PHILOSOPHICAL TRANSACTIONS.

XV.—*Researches in Physical Astronomy.* *By* J. W. Lubbock, *Esq. V.P. and Treas. R.S.*

Read May 19, 1831.

On the Theory of the Moon.

THE method pursued by Clairaut in the solution of this important problem of Physical Astronomy, consists in the integration of the differential equations furnished by the principles of dynamics, upon the hypothesis that in the gravitation of the celestial bodies the force varies inversely as the square of the distance, and in which the true longitude of the moon is the independent variable; the time is thus obtained in terms of the true longitude, and by the reversion of series the longitude is afterwards obtained in terms of the time, which is necessary for the purpose of forming astronomical tables. But while on the one hand this method possesses the advantage, that the disturbing function can be developed with somewhat greater facility in terms of the true longitude of the moon than in terms of the mean longitude, yet on the other hand, the differential equations in which the true longitude is the independent variable are far more complicated than those in which the time is the independent variable. The latter equations are used in the planetary theory; so that the method of Clairaut has the additional inconvenience, that while the lunar theory is a particular case of the problem of the three bodies, one system of equations is used in this case, and another in the case of the planets.

The method of Clairaut has been adopted, however, by Mayer, by Laplace, and by M. Damoiseau. The last-mentioned author has arranged his results with remarkable clearness, so that any part of his processes may be easily verified by any one who does not shrink from this gigantic undertaking; and the immense labour which this method requires, when all sensible quantities are retained, may be seen in his invaluable memoir. Mr. Brice Bronwin has recently communicated to the Society a lunar theory, in which the same method is adopted.

Having reflected much upon the difficulties of this problem, I am led to believe that the integration of the differential equations in which the time is the independent variable, is at least as easy as the method hitherto, I think, solely employed, and I now have the honour to submit to the Society a lunar theory founded upon this integration, which is in fact merely an extension of the equations given in my Researches in Physical Astronomy, already printed, by embracing those terms which, in consequence of the magnitude of the eccentricity of the moon's orbit, are sensible; and the suppression of those, on the other hand, which are insensible on account of the great distance of the sun, the disturbing body. By means of the Table which I have given (Table II.), the developments may all be effected at once with the greatest facility.

The first column contains the indices, which I have employed to distinguish the inequalities. The numbers in the second column are the indices affixed by M. Damoiseau, in the Mém. sur la Théor. de la Lune, p. 547. to the inequalities of longitude.

$t^{*} = nt - n_{\prime}t, \quad x = cnt - \varpi, \quad z = n_{\prime}t - \varpi_{\prime}, \quad y = gnt - \nu.$

0	..	0	21	45	$2t - 3x$	42	73	$2t - 3x - z$
1	30	$2t$ †	22	46	$2t + 3x$	43	..	$2t + 3x + z$
2	1	x	23	21	$2x + z$	44	26	$3x - z$
3	31	$2t - x$ ‡	24	53	$2t - 2x - z$	45	..	$2t - 3x + z$
4	32	$2t + x$	25	54	$2t + 2x + z$	46	..	$2t + 3x - z$
5	16	z §	26	20	$2x - z$	47	..	$2x + 2z$
6	33	$2t - z$	27	51	$2t - 2x + z$	48	75	$2t - 2x - 2z$
7	34	$2t + z$	28	52	$2t + 2x - z$	49	..	$2t + 2x + 2z$
8	2	$2x$	29	23	$x + 2z$	50	..	$2x - 2z$
9	35	$2t - 2x$	30	59	$2t - x - 2z$	51	..	$2t - 2x + 2z$
10	36	$2t + 2x$	31	..	$2t + x + 2z$	52	..	$2t + 2x - 2z$
11	19	$x + z$	32	22	$x - 2z$	53	..	$x + 3z$
12	41	$2t - x - z$	33	61	$2t - x + 2z$	54	..	$2t - x - 3z$
13	42	$2t + x + z$	34	60	$2t + x - 2z$	55	..	$2t + x + 3z$
14	18	$x - z$	35	..	$3z$	56	..	$x - 3z$
15	39	$2t - x + z$	36	..	$2t - 3z$	57	..	$2t - x + 3z$
16	40	$2t + x - z$	37	..	$2t + 3z$	58	..	$2t + x - 3z$
17	17	$2z$	38	9	$4x$	59	..	$4z$
18	43	$2t - 2z$	39	67	$2t - 4x$	60	..	$2t - 4z$
19	44	$2t + 2z$	40	..	$2t + 4x$	61	..	$2t + 4z$
20	4	$3x$	41	27	$3x + z$	62	3	$2y$

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* Inconvenience arises from using the letter t in this acceptation. I have done so in order to conform to the notation of M. Damoiseau. † Variation. ‡ Evection. § Annual Equation.

63	37	$2t-2y$	105	84	$t+z$	146	....	y
64	38	$2t+2y$	106	85	$t-2x$	147	....	$2t-y$
65	5	$x-2y$	107	86	$t+2x$	148	....	$2t+y$
66	6	$x+2y$	108	91	$t-x-z$	149	....	$x-y$
67	49	$2t-x-2y$	109	92	$t+x+z$	150	....	$x+y$
68	47	$2t-x+2y$	110	89	$t-x+z$	151	....	$2t-x-y$
69	48	$2t+x-2y$	111	....	$t+x-z$	152	....	$2t-x+y$
70	50	$2t+x+2y$	112	....	$t-2z$	153	....	$2t+x-y$
71	24	$z-2y$	113	....	$t+2z$	154	....	$2t+x+y$
72	25	$z+2y$	114	....	$t-2y$	155	....	$z-y$
73	57	$2t-z-2y$	115	....	$t+2y$	156	....	$z+y$
74	56	$2t-z+2y$	116	100	$3t$	157	....	$2t-z-y$
75	55	$2t+z-2y$	117	101	$3t-x$	158	....	$2t-z+y$
76	58	$2t+z+2y$	118	102	$3t+x$	159	....	$2t+z-y$
77	7	$2x-2y$	119	103	$3t-z$	160	....	$2t+z+y$
78	8	$2x+2y$	120	104	$3t+z$	161	....	$2x \quad y$
79	65	$2t-2x-2y$	121	....	$3t-2x$	162	....	$2x+y$
80	63	$2t-2x+2y$	122	....	$3t+2x$	163	....	$2t-2x-y$
81	64	$2t+2x-2y$	123	....	$3t-x-z$	164	....	$2t-2x+y$
82	..	$2t+2x+2y$	124	....	$3t+x+z$	165	....	$2t+2x-y$
83	..	$x+z-2y$	125	....	$3t-x+z$	166	....	$2t+2x+y$
84	..	$x+z+2y$	126	....	$3t+x-z$	167	....	$x+z-y$
85	..	$2t-x-z-2y$	127	....	$3t-2z$	168	....	$x+z+y$
86	..	$2t-x-z+2y$	128	....	$3t+2z$	169	....	$2t-x-z-y$
87	..	$2t+x+z-2y$	129	....	$3t-2y$	170	....	$2t-x-z+y$
88	..	$2t+x+z+2y$	130	....	$3t+2y$	171	....	$2t+x+z-y$
89	..	$x-z-2y$	131	120	$4t$	172	....	$2t+x+z+y$
90	..	$x-z+2y$	132	121	$4t-x$	173	....	$x-z-y$
91	..	$2t-x+z-2y$	133	122	$4t+x$	174	....	$x-z+y$
92	..	$2t-x+z+2y$	134	123	$4t-z$	175	....	$2t-x+z-y$
93	..	$2t+x-z-2y$	135	124	$4t+z$	176	....	$2t-x+z+y$
94	..	$2t+x-z+2y$	136	125	$4t-2x$	177	....	$2t+x-z-y$
95	..	$2z-2y$	137	126	$4t+2x$	178	....	$2t+x-z+y$
96	..	$2z+2y$	138	131	$4t-x-z$	179	....	$2z-y$
97	..	$2t-2z-2y$	139	....	$4t+x+z$	180	....	$2z+y$
98	..	$2t-2z+2y$	140	129	$4t-x+z$	181	....	$2t-2z-y$
99	..	$2t+2z-2y$	141	....	$4t+x-z$	182	....	$2t-2z+y$
100	..	$2t+2z+2y$	142	....	$4t-2z$	183	....	$2t+2z-y$
101	80	t*	143	....	$4t+2z$	184	....	$2t+2z+y$
102	81	$t-x$	144	127	$4t-2y$	185	....	$t-y$
103	82	$t+x$	145	....	$4t+2y$	186	....	$t+y$
104	83	$t-z$						

$$\cos 2t \cos 2t = \underset{[131]}{\frac{1}{2}\cos 4t} + \underset{[0]}{\frac{1}{2}}$$

$$\cos 2t \cos x = \underset{[4]}{\frac{1}{2}\cos(2t+x)} + \underset{[-3]}{\frac{1}{2}\cos(-2t+x)}$$

Hence the multiplication of cos 2 t by cos 2 t produces the arguments 131 and 0, similarly the multiplication of cos x by cos 2 t produces the arguments 4 and − 3; proceeding in this way the following Table was formed, by writing down the indices instead of the arguments themselves.

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* Parallactic inequality.

TABLE I.

Showing the arguments which result from the combination of the arguments 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 17, 20, 35, 62, 101, 146 and 147, with the arguments 1, 2, 3, &c. by addition and subtraction.

0	1	2	3	4	5	6	7	8	9	10	11	12	14	17	20	35	62	101	146	147	
1	131	4	132	133	7	134	135	10	136	137	13	138	16	19	22	37	64	116	148	……	1
	0	3	2	−2	6	5	−5	9	8	−8	12	11	15	18	21	36	63	101	147	146	
2	4	8	1	10	11	16	13	20	3	22	23	6	26	29	38	53	66	103	150	153	2
	−3	0	−9	−1	14	−12	−15	−2	−21	−4	−5	−24	5	32	−8	56	65	−102	149	−151	
3	132	1	136	131	15	138	140	4	……	133	7	……	6	33	10	57	68	117	152	……	3
	−2	9	0	−8	12	−14	−11	21	2	−20	24	5	27	30	39	54	67	102	151	−149	
4	133	10	131	137	13	141	139	22	132	……	25	134	28	31	40	55	70	118	154	……	4
	2	1	8	0	16	11	14	3	20	−2	6	23	7	34	9	58	69	103	153	150	
5	7	11	15	13	17	1	19	23	27	25	29	3	2	35	41	59	72	105	156	159	5
	−6	−14	−12	−16	0	−18	−1	−26	−24	−28	−2	−30	−32	−5	−44	−17	71	−104	155	−157	
6	134	16	138	141	1	142	131	28	……	……	4	……	34	7	46	19	74	119	158	……	6
	−5	12	14	−11	18	0	−17	24	26	−23	30	2	3	36	42	60	73	104	157	−155	
7	135	13	140	139	19	131	143	25	……	……	31	132	4	37	43	61	76	120	160	……	7
	5	15	11	−14	1	17	0	27	23	−26	3	29	33	6	45	18	75	105	159	156	
8	10	20	4	22	23	28	25	38	1	……	41	16	44	47	……	……	78	107	162	165	8
	−9	2	−21	−3	26	−24	27	0	−39	−1	14	−42	11	50	……	……	77	−106	161	−163	
9	136	3	……	132	27	……	……	1	……	131	15	……	12	51	……	……	80	121	164	……	9
	−8	21	−2	−20	24	−26	−23	39	0	−38	42	−14	45	48	……	……	79	106	163	−161	
10	137	22	133	……	25	……	…..	40	131	……	43	141	46	49	……	……	82	122	166	……	10
	8	4	20	2	28	23	26	1	38	0	16	41	13	52	……	……	81	107	165	162	
11	13	23	7	25	29	4	31	41	15	43	47	1	8	53	……	……	84	109	168	171	11
	−12	5	−24	−6	2	−30	−3	−14	−42	−16	0	−48	17	14	……	……	83	−108	167	−169	
12	138	6	……	134	3	……	132	16	……	141	1	……	18	15	……	……	86	123	170	……	12
	−11	24	−5	−23	30	−2	−29	42	14	−41	48	0	9	54	……	……	85	108	169	−167	
13	139	25	135	……	31	133	……	43	140	…..	49	131	10	55	……	……	88	124	172	……	13
	11	7	23	5	4	29	2	15	41	−14	1	47	19	16	……	……	87	109	171	168	
14	16	26	6	28	2	34	4	44	12	46	8	18	50	11	……	……	90	111	174	177	14
	−15	−5	−27	−7	32	−3	−33	−11	−45	−13	−17	−9	0	56	……	……	89	−110	173	−175	
15	140	7	……	135	33	132	……	13	……	139	19	136	1	57	……	……	92	125	176	……	15
	−14	27	5	−26	3	−32	−2	45	11	−44	9	17	51	12	……	……	91	110	175	−173	
16	141	28	134	……	4	……	133	46	138	……	10	142	52	13	……	……	94	126	178	……	16
	14	6	26	−5	34	2	32	12	44	−11	18	8	1	58	……	……	93	111	177	174	
17	19	29	33	31	35	7	37	47	51	49	53	15	11	59	……	……	96	113	180	183	17
	−18	−32	−30	34	5	−36	−6	−50	−48	−52	−14	−54	−56	0	……	……	95	−112	179	−181	
18	142	34	……	……	6	……	134	52	……	……	16	……	58	1	…..	……	92	127	182	……	18
	−17	30	32	−29	36	−5	−35	48	50	−47	54	14	12	60	……	……	97	112	181	−179	
19	143	31	……	……	37	135	……	49	……	……	55	140	13	61	……	……	100	128	184	……	19
	17	33	29	−32	7	35	5	51	47	−50	15	53	57	1	……	……	99	113	183	180	
20	22	38	10	40	41	……	……	……	……	…..	……	……	……	……	……	……	……	……	……	……	20
	−21	8	−39	−9	44	……	……	……	……	……	……	……	……	……	……	……	……	……	……	……	
21	……	9	……	136	45	……	……	……	……	……	……	……	……	……	……	……	……	……	……	……	21
	−20	39	−8	−38	42	……	……	……	……	…..	……	……	……	……	……	……	……	……	……	……	

TABLE I. (Continued.)

0	1	2	3	4	5	6	7	8	9	10	11	12	14	17	20	35	62	101	146	147	
22	......	40	......	......	43	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	22
	20	10	38	8	46	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
23	25	41	13	43	47	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	23
	− 24	11	− 42	− 12	8	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
24	......	12	......	138	9	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	24
	− 23	42	− 11	− 41	48	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
25	......	43	139	......	49	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	25
	23	13	41	11	10	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
26	28	44	16	46	8	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	26
	− 27	14	− 45	− 15	50	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
27	......	15	......	140	51	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	27
	− 26	45	− 14	− 44	9	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
28	......	46	141	......	10	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	28
	26	16	44	14	52	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
29	31	47	19	49	53	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	29
	− 30	17	− 48	− 18	11	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
30	......	18	......	142	12	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	30
	− 29	48	− 17	− 47	54	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
31	......	49	143	......	55	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	31
	29	19	47	17	13	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
32	34	50	18	52	14	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	32
	− 33	− 17	− 51	− 19	56	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
33	......	19	......	143	57	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	33
	− 32	51	17	− 50	15	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
34	......	52	142	......	16	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	34
	32	18	50	− 17	58	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
35	37	53	57	58	59	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	35
	− 36	− 56	− 54	− 58	17	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
36	......	58	......	......	18	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	36
	− 35	54	56	− 53	60	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
37	......	55	......	......	61	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	37
	35	57	53	− 56	19	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
62	64	66	68	70	72	74	76	78	80	82	84	86	90	96	......	......	...	115	...	148	62
	− 63	− 65	− 67	− 69	− 71	− 73	− 75	− 77	− 79	− 81	− 83	− 85	−89	− 95	......	......	0	−114	146	......	
63	144	69	......	......	75	......	......	81	......	......	87	......	93	99	......	......	1	129	147	......	63
	− 62	67	65	− 66	73	71	− 72	79	77	− 78	85	83	91	97	......	......	...	114	...	−146	
64	145	70	......	......	76	......	......	82	......	......	88	......	94	100	......	......	...	130	...	......	64
	62	68	66	− 65	74	72	− 71	80	78	− 77	86	84	92	98	......	......	1	115	148	......	
65	69	77	63	81	83	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	65
	− 68	− 62	77	− 64	89	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
66	70	78	64	82	84	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	66
	− 67	62	− 79	− 63	90	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
67	......	63	......	144	91	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	67
	− 66	79	− 62	− 78	85	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
68	......	64	......	145	92	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	68
	− 65	80	62	− 77	86	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
69	......	81	144	......	87	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	69
	65	63	77	− 62	93	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	

TABLE I. (Continued.)

0	1	2	3	4	5	6	7	8	9	10	11	12	14	17	20	35	62	101	146	147	
70	......	82	145	......	88	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	70
	66	64	78	62	94	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	
71	75	83	91	87	95	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	71
	− 74	− 90	− 86	− 94	− 62	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	
72	76	84	92	88	96	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	72
	− 73	− 89	− 85	− 93	62	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	
73	......	93	......	......	63	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	73
	− 72	85	89	− 84	97	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	
74	......	94	......	......	64	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	74
	− 71	86	90	− 83	98	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	
75	......	87	......	......	99	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	75
	71	91	83	− 90	63	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	
76	......	88	......	......	100	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	76
	72	92	84	− 89	64	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	
101	116	103	117	118	105	119	120	107	121	122	109	123	111	113	...	...	115	1	186	......	101
	−101	102	−102	−103	104	−104	−105	106	−106	−107	108	−108	110	112	...	...	114	0	185	−185	
102	117	101	121	116	110	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	102
	−103	106	−101	−107	108	......	......	......	......	......	......	......	......	......	...	...	......	......	...	.. ...	
103	118	107	116	122	109	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	103
	−102	101	−106	−101	111	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	
104	119	111	123	126	101	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	104
	−105	108	−110	−109	112	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	
105	120	109	125	124	113	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	105
	−104	110	−108	−111	101	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	
116	......	118	......	......	120	......	......	122	......	......	124	......	126	128	...	...	130	131	...	......	116
	101	117	103	102	119	105	104	121	107	106	123	109	125	127	...	...	129	1	...	186	
117	......	116	......	......	125	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	117
	102	121	101	106	123	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	
118	......	122	......	......	124	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	118
	103	116	107	101	126	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	
119	......	126	......	......	116	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	119
	104	123	111	108	127	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	
120	......	124	......	......	128	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	120
	105	125	109	110	116	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	
131	......	133	......	......	135	......	......	137	......	......	139	......	141	143	...	...	145	......	...	......	131
	1	132	4	3	134	7	6	136	10	9	138	13	140	142	...	...	144	116	...	148	
132	......	131	......	......	140	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	132
	3	136	1	9	138	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	
133	......	137	......	......	139	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	133
	4	131	10	1	141	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	
134	......	141	......	......	131	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	134
	6	138	16	12	142	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	
135	......	139	......	......	143	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	135
	7	140	13	15	131	......	......	......	......	......	......	......	......	......	...	...	......	......	...	......	
146	148	150	152	154	156	158	160	162	164	166	168	170	174	180	...	...	......	186	62	1	146
	−147	−149	−151	−153	−155	−157	−159	−161	163	−165	−167	−169	−173	−179	...	...	−146	−185	0	− 63	
147	......	153	......	......	159	......	......	164	......	......	171	......	177	183	...	...	148	......	1	144	147
	−146	151	149	−150	157	155	−156	163	161	−162	169	167	175	181	...	...	......	185	63	0	

TABLE I. (Continued.)

0	1	2	3	4	5	6	7	8	9	10	11	12	14	17	20	35	62	101	146	147	
148	 146	154 152	 150	...... −149	160 158	 156	...... −155	166 164	 162	...... −161	172 170	 168	178 176	184 182	... ...	... ...	 147	 186	64 1	131 62	148
149	153 −152	161 −146	147 −164	165 −148	167 173	....	....	....	....	....	....	....	....	....	... ...	... ...	....	....	... ...	....	149
150	154 −151	162 146	148 −163	166 −147	168 174	....	....	....	....	....	....	....	....	....	... ...	... ...	....	....	... ...	....	150
151	...... −150	147 163	...... −146	...... −162	175 169	....	....	....	....	....	....	....	....	....	... ...	.. ...	....	....	... ...	....	151
152	...... −149	148 164	 146	...... −161	176 170	....	....	....	....	....	....	....	....	....	... ...	... ...	....	....	... ...	....	152
153	 149	165 147	 161	...... −146	171 177	....	....	....	....	....	....	....	....	....	... ...	... ...	....	....	... ...	....	153
154	 150	166 148	 162	 146	172 178	....	....	....	....	....	....	....	....	....	... ...	... ...	....	....	... ...	....	154
155	159 −158	167 −174	175 −170	...... −178	179 −146	....	....	....	....	....	....	....	....	....	... ...	... ...	....	....	... ...	....	155
156	160 −157	168 −173	176 −169	172 −177	180 146	....	....	....	....	....	....	....	....	....	... ...	... ...	....	....	... ...	....	156
157	...... −156	177 169	 173	...... −168	147 181	....	....	....	....	....	....	....	....	....	... ...	... ...	....	....	... ...	....	157
158	...... −155	178 170	 174	...... −167	148 182	....	....	....	....	....	....	....	....	....	... ...	... ...	....	....	... ...	....	158
159	 155	171 175	 167	...... −174	183 147	....	....	....	....	....	....	....	....	....	... ...	... ...	....	....	... ...	....	159
160	 156	172 176	 168	...... −173	184 148	....	....	....	....	....	....	....	....	....	... ...	... ...	....	....	... ...	....	160

	38	59	
1	40 39	61 60	1

TABLE II.

Showing the arguments which, by their combination with the arguments 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 17, 20, 35, 62, 101, 146, 147, produce the arguments, 12, 3, &c. in the left hand column. This Table is formed from the preceding, by making the numbers in the left hand column in that Table change places with the rest. A full stop is placed after the figure where it does not occupy the same *cell* as in the preceding Table.

0	1	2	3	4	5	6	7	8	9	10	11	12	14	17	20	35	62	101	146	147	
1	0 131	3 4	2 132	133. − 2	6 7	5	...... − 5	9 10	8 ...	...... − 8	12 13	11	15 16	18 19	... ...	... ...	63 64	101 116	147 148	146 ...	1
2	4. − 3	0 8	− 9. 1	10. − 1	14 11	16. − 12	− 15. 13	...... − 2	... 3	...... − 4	...... − 5	 6	5 ...	....	... ...	... ...	... ...	... ...	... ...	... ...	2
3	132. − 2	9 1	0	131. − 8	12 15	...... − 14	...... − 11	 4	2 ...	....	 7	5	... 6	....	... ...	... ...	... ...	... ...	... ...	... ...	3
4	2 133	1 10	8 131	0	16 13	11	14	3	... ...	...... − 2	6	....	7 ...	....	... ...	... ...	... ...	... ...	... ...	... ...	4
5	7. − 6	− 14. 11	15. − 12	− 16. 13	0 17	− 18. 1	19. − 1	....	... ...	....	...... − 2	 3	... 2	...... − 5	... ...	... ...	... ...	... ...	... ...	... ...	5
6	134. − 5	12 16	14	...... − 11	18 1	....	131. − 17	....	... ...	....	 4	2	3 ...	 7	... ...	... ...	... ...	... ...	... ...	... ...	6
7	5 135	15 13	11	...... − 14	1 19	17 131	....	....	... ...	....	3	....	... 4	6	... ...	... ...	... ...	... ...	... ...	... ...	7
8	10. − 9	2 20	− 21. 4	22. − 3	26 23	....	....	....	... 1	...... − 1	14	 16	11 ...	....	... − 2	... ...	... ...	... ...	... ...	... ...	8
9	...... − 8	21 3	...... − 2	132. − 20	24 27	....	....	 1	... ...	 131	 15	...... −14	... 12	....	... 4	... ...	... ...	... ...	... ...	.. ...	9
10	8	4 22	20 133	2	28 25	....	....	1	... 131	....	16	....	13 ...	....	3 ...	... ...	... ...	... ...	... ...	... ...	10
11	13. − 12	5 23	− 24. 7	25. − 6	2 29	 4	...... − 3	...... − 14	... 15	...... − 16	....	 1	17 8	14	... ...	... ...	... ...	... ...	... ...	... ...	11
12	...... − 11	24 6	...... − 5	134. − 23	30 3	...... − 2	....	 16	14 ...	....	 1	....	9 18	 15	... ...	... ...	... ...	... ...	... ...	... ...	12
13	11	7 25	23 135	5	4 31	....	2	15	... ...	...... − 14	1	 131	19 10	16	... ...	... ...	... ...	... ...	... ...	... ...	13
14	16. − 15	26. − 5	− 27. 6	28. − 7	32 2	...... − 3	 4	...... − 11	... 12	...... − 13	− 17. 8	18. − 9	... ...	 11	... ...	... ...	... ...	... ...	... ...	... ...	14
15	...... − 14	27 7	5	135. − 26	3 33	....	...... − 2	 13	11 ...	....	9 19	17	... 1	12	... ...	... ...	... ...	... ...	... ...	... ...	15
16	14	6 28	26 134	...... − 5	34 4	2	....	12	... ...	...... − 11	18 10	8	1 ...	 13	... ...	... ..	... ...	. . ...	... ...	... ...	16
17	19. − 18	− 32. 29	33 − 30	− 34. 31	5 35	 7	...... − 6	....	... ...	....	...... − 14	 15	... 11	0	... ...	... − 5	... ...	... ...	.. ...	... ...	17
18	...... − 17	30 34	32	...... − 29	36 6	...... − 5	....	....	... ...	....	 16	14	12 ...	 1	... ...	... 7	... ...	... ...	... ...	... ...	18
19	17	33 31	29	...... − 32	7 37	....	5	....	... ...	....	15	....	... 13	1	... ...	6 ...	... ...	... ..	... ...	... ...	19
20	22. − 21	8	 10	...... − 9	... ...	....	....	2	... 4	...... − 3	....	....	... ...	....	0 ...	... ...	... ...	... ...	... ...	... ...	20

Table II. (Continued.)

	1	2	3	4	5	6	7	8	9	10	11	12	14	17	20	35	62	101	146	147	
21	......	......	......	......	...	......	......	......	......	......	...	......	......	...	...	...	...	...	...	...	21
	– 20	9	– 8	......	...	......	......	3	– 2	......	...	......	......	...	1	...	...	...	...	...	
22	20	10	......	8	...	......	......	4	......	2	...	......	......	...	1	...	...	...	...	...	22
	......	......	......	......	...	......	......	......	......	......	...	......	......	...	...	...	...	...	...	...	
23	25.	11	......	......	8	......	......	5	......	......	2	......	......	...	...	...	...	...	...	...	23
	– 24	......	13	– 12	...	10	– 9	......	7	– 6	...	4	......	...	...	...	...	...	...	...	
24	......	......	......	......	...	......	......	......	......	......	...	......	......	...	...	...	...	...	...	...	24
	– 23	12	– 11	......	9	– 8	......	6	– 5	......	3	– 2	......	...	...	...	...	...	...	...	
25	23	13	......	11	10	......	8	7	......	5	4	......	......	...	...	...	...	...	...	..	25
	......	......	......	......	...	......	......	......	......	......	...	......	......	...	...	...	...	...	...	...	
26	28.	14	......	......	...	......	......	......	......	......	...	......	2	...	...	...	...	...	...	...	26
	– 27	......	16	– 15	8	– 9	10	– 5	6	– 7	...	......	......	...	...	...	...	...	...	...	
27	......	......	......	......	9	......	......	......	5	......	...	......	......	...	..	...	...	...	...	...	27
	– 26	15	– 14	......	...	......	8	7	......	......	...	......	3	...	...	...	...	...	...	...	
28	26	16	......	14	...	8	......	6	......	......	...	......	4	...	...	...	...	...	...	...	28
	......	......	......	......	10	......	......	......	......	– 5	...	......	......	...	...	...	...	...	...	...	
29	31.	17	......	......	11	......	......	......	......	......	5	......	......	2	...	...	...	...	...	...	29
	– 30	......	19	– 18	...	13	– 12	......	......	......	...	7	......	...	...	...	...	...	..	...	
30	......	......	18.	......	...	......	......	......	......	......	...	......	......	...	...	...	...	...	...	...	30
	– 29	18	– 17	......	12	– 11	......	......	......	......	6	– 5	......	3	...	...	...	...	...	...	
31	29	19	......	17	13	......	11	......	......	......	7	......	......	4	..	...	...	...	...	...	31
	......	......	......	......	...	......	......	......	......	......	...	......	......	...	...	...	...	...	...	...	
32	34.	......	......	......	...	......	......	......	......	......	...	......	......	...	...	...	...	...	...	...	32
	– 33	– 17	18	– 19	14	– 15	16	......	......	......	...	......	– 5	2	...	...	...	...	...	...	
33	......	......	17	......	15	......	......	......	......	......	...	......	......	3	...	...	...	...	...	...	33
	– 32	19	......	......	...	......	– 14	......	......	......	...	......	7	...	...	...	...	...	...	...	
34	32	18	......	......	...	14	......	......	......	......	...	......	6	...	...	...	...	...	...	...	34
	......	......	......	17	16	......	......	......	......	......	...	......	......	4	...	...	...	...	...	...	
35	37.	......	......	......	17	......	......	......	......	......	...	......	......	5	...	...	...	...	...	...	35
	– 36	......	......	......	...	19	– 18	......	......	......	...	......	......	...	...	...	...	...	...	...	
36	......	......	......	......	...	......	......	......	......	......	...	......	......	...	...	...	...	...	...	...	36
	– 35	......	......	......	18	– 17	......	......	.....	......	...	......	......	6	...	1	...	...	...	...	
37	35	......	......	......	19	......	17	......	......	......	...	......	......	7	...	1	...	...	...	...	37
	......	......	......	......	...	......	......	......	.. ...	......	...	......	......	...	...	...	...	...	...	...	
38	......	20	......	......	...	......	......	8	......	......	...	......	......	...	2	...	...	...	...	...	38
	......	......	22	– 21	...	......	......	......	10	– 9	...	......	......	...	...	...	...	...	...	...	
39	......	......	......	......	...	......	......	......	......	......	...	......	......	...	...	...	...	...	...	...	39
	.....	21	– 20	......	...	......	......	9	– 8	......	...	......	......	...	3	...	...	...	...	...	
40	......	22	......	20	...	......	......	10	......	......	...	......	......	...	4	...	...	...	...	...	40
	......	......	......	......	...	......	......	......	......	......	...	......	......	...	...	...	...	...	...	...	
41	......	23	......	......	20	......	......	11	......	......	8	......	......	..	5	...	...	...	...	...	41
	......	......	25	– 24	...	......	......	......	13	– 12	...	10	......	...	...	...	...	...	...	...	
42	......	......	......	......	...	......	......	......	......	......	...	......	......	...	...	...	...	...	...	...	42
	......	24	– 23	......	21	......	......	12	– 11	......	9	– 8	......	...	6	...	...	...	...	...	
43	......	25	......	23	22	......	......	13	......	11	10	......	......	...	7	...	...	...	...	...	43
	......	......	......	......	...	......	......	......	......	......	...	......	......	...	...	...	...	...	...	...	
44	......	26	......	......	...	... ..	......	14	......	......	...	......	8	...	...	...	...	...	...	...	44
	.. ...	......	28	– 27	20	......	......	......	16	– 15	...	......	......	...	–5	...	...	...	..	...	

TABLE II. (Continued.)

0	1	2	3	4	5	6	7	8	9	10	11	12	14	17	20	35	62	101	146	147	
45	....	 27	...... – 26	....	21	... ...	... ...	 15	...... – 14	....	... ...	....	 9	... ...	... 7	... ...	... ...	... ...	... ...	... ...	45
46	....	28	....	26	 22	... ...	... ...	16	....	14	... ...	....	10	... ...	6 ...	... ...	... ...	... ..	... ...	... ...	46
47	....	29	 31	...... – 30	23	... ...	... ...	17	 19	...... – 18	11 ...	 13	....	8 ...	... ...	... ...	... ...	... ...	... ...	... ...	47
48	....	 30	...... – 29	....	 24	... ...	... ...	 18	...... – 17	....	... 12	...... – 11	....	... 9	... ...	... ...	... ...	... ...	... ...	... ...	48
49	....	31	....	29	25	... ...	... ...	19	....	17	13 ...	....	....	10 ..	... ...	... ...	... ...	... ...	... ...	... ...	49
50	....	32	 34	...... – 33	 26	... ...	... ...	... .. – 17	 18	...... – 19	... ...	....	14	... 8	... ...	... ...	... ...	... ...	... ...	... ...	50
51	....	 33	...... – 32	....	27	... ...	... ...	 19	17	....	... ...	....	 15	9 ...	... ...	... ...	... ...	... ...	.. ...	... ...	51
52	....	34	....	32	 28	... ...	... ...	18	....	...... – 17	... ...	....	16	... 10	... ...	... ...	... ...	... ...	... ...	... ...	52
53	....	35	 37	...... – 36	29	... ...	... ...	....	....	....	17 ...	 19	....	11 ...	... ...	2 ...	... ...	... ...	... ...	... ...	53
54	....	 36	...... – 35	....	 30	... ...	... ...	....	....	....	... 18	...... – 17	....	... 12	... ...	... 3	... ...	... ...	... ...	... ...	54
55	....	37	....	....	31	... ...	... ...	....	....	....	19 ...	....	....	13 ...	... ...	4 ...	... ...	... ...	... ...	... ...	55
56	....	...... – 35	 36	...... – 37	 32	... ...	... ...	....	....	....	... ...	....	...... – 17	... 14	... ...	... 2	... ...	... ...	... ...	... ...	56
57	....	 37	35	....	33	... ...	... ...	....	....	....	... ...	....	 19	15 ...	... ...	3 ...	... ...	... ...	... ...	... ...	57
58	....	36	....	35. – 35	 34	... ...	... ...	....	....	....	... ...	....	18	... 16	... ...	... 4	... ...	... ...	... ...	... ...	58
59	....	....	....	....	35	... ...	... ...	....	....	....	... ...	....	....	17 ...	... ...	5 ...	... ...	... ...	... ...	... ...	59
60	....	....	....	....	 36	... ...	... ...	....	....	....	... ...	....	....	... 18	... ...	... 6	... ...	... ...	... ...	... ...	60
61	....	....	....	....	37	... ...	... ...	....	....	....	... ...	....	....	19 ...	... ..	7 ...	... ...	... ...	... ...	... ...	61
62	64. – 63	66. – 65	68. – 67	70. – 69	72. – 71	... ...	... ...	....	....	....	... ...	....	....	... ...	... ...	.. ...	... ...	... ...	146 ...	... 148	62
63	...... – 62	67 69	65	...... – 66	73 75	... ...	... ...	....	....	....	... ...	....	....	... ...	... ...	... ...	... 1	... ...	... 147	... –146	63
64	62	68 70	66	...... – 65	74 76	... ...	... ...	....	....	....	... ...	....	....	... ...	... ...	... ...	1 ...	... ...	148 ...	... ...	64
65	69. – 68	...... – 62	 63	...... – 64	....	... ...	... ...	....	....	....	... ...	....	....	... ...	... ...	... ...	... 2	... ...	... ...	... ...	65
66	70. – 67	62	 64	...... – 63	....	... ...	... ...	....	....	....	... ...	....	....	... ..	... ...	... ...	2 ...	... ...	... ...	... ...	66
67	76 – 66	 63	...... – 62	....	....	... ...	... ...	....	....	....	... ...	....	....	... ...	... ...	... ...	... 3	... ...	... ...	... ...	67
68	...... – 65	 64	62	....	....	... ...	... ...	....	....	....	... ...	....	....	... ...	.. ...	... ...	3 ...	... ...	... ...	... ...	68

TABLE II. (Continued.)

0	1	2	3	4	5	6	7	8	9	10	11	12	14	17	20	35	62	101	146	147	
69	65	63	...	......	......	......	......	......	......	......	......	......	......	...	...	...	...	...	...	...	69
	......	...	...	− 62	......	......	......	......	......	......	......	......	......	...	...	...	4	...	...	...	
70	66	64	...	62	......	......	......	......	......	......	......	......	......	...	...	...	4	...	...	...	70
	......	...	...	......	......	......	......	......	......	......	......	......	......	...	...	...	...	...	...	...	
71	75.	...	...	......	......	......	......	......	......	......	......	......	......	...	...	...	...	...	...	...	71
	− 74	...	...	......	− 62	63	− 64	......	......	......	......	......	......	...	...	...	5	...	...	...	
72	76.	...	...	......	62	......	......	......	......	......	......	......	......	...	...	...	5	...	...	...	72
	− 73	...	...	......	......	64	− 63	......	......	......	......	......	......	...	...	...	...	...	...	...	
73	......	...	...	......	......	......	......	......	......	......	......	......	......	...	...	...	...	...	...	...	73
	− 72	...	...	......	63	− 62	......	......	......	......	......	......	......	...	...	...	6	...	...	...	
74	......	...	...	......	......	62	......	......	......	......	......	......	......	...	...	...	6	...	...	...	74
	− 71	...	...	......	64	......	......	......	......	......	......	......	......	...	...	...	...	...	...	...	
75	71	...	...	......	63	......	......	......	......	......	......	......	......	...	...	...	...	...	...	...	75
	......	...	...	......	......	......	− 62	......	......	......	......	......	......	...	...	...	7	...	...	...	
76	72	...	...	......	64	......	62	......	......	......	......	......	......	...	...	...	7	...	...	...	76
	......	...	...	......	......	......	......	......	......	......	......	......	......	...	...	...	...	...	...	...	
77	......	65	69.	......	......	......	......	......	......	......	......	......	......	...	...	...	...	...	...	...	77
	......	...	65	− 68	......	......	......	− 62	63	− 64	......	......	......	...	...	...	8	...	...	...	
78	......	66	...	......	......	......	......	62	......	......	......	......	......	...	...	...	8	...	...	...	78
	......	...	70	− 67	......	......	......	......	64	− 63	......	......	......	...	...	...	...	...	...	...	
79	......	...	...	......	......	......	......	......	......	......	......	......	......	...	...	...	...	...	...	...	79
	......	67	−66	......	......	......	......	63	− 62	......	......	......	......	...	...	...	9	...	...	...	
80	......	...	...	......	......	......	......	......	62	......	......	......	......	...	...	...	9	...	...	...	80
	......	68	...	......	......	......	......	64	......	......	......	......	......	...	...	...	...	...	...	...	
81	......	69	...	65	......	......	......	63	......	......	......	......	......	...	...	...	...	...	...	...	81
	......	...	...	......	......	......	......	......	......	− 62	......	......	......	...	...	...	10	...	...	...	
82	......	70	...	66	......	......	......	64	......	62	......	......	......	...	...	...	10	...	...	...	82
	......	...	...	......	......	......	......	......	......	......	......	......	......	...	...	...	...	...	...	...	
83	......	71	...	......	65	......	......	......	......	......	......	......	......	...	...	...	...	...	...	...	83
	......	...	75	− 74	......	......	......	......	......	......	− 62	63	......	...	...	...	11	...	...	...	
84	......	72	...	......	66	......	......	......	......	......	62	......	......	...	...	...	11	...	...	...	84
	......	...	76	− 73	......	......	......	......	......	......	......	64	......	...	...	...	...	...	...	...	
85	......	...	...	......	......	......	......	......	......	......	......	......	......	...	...	...	...	...	...	...	85
	......	73	−72	......	67	......	......	......	......	......	63	− 62	......	...	...	...	12	...	...	...	
86	......	...	...	......	......	......	......	......	......	......	......	62	......	...	...	...	12	...	...	...	86
	......	74	−71	......	68	......	......	......	......	......	64	......	......	...	...	...	...	...	...	...	
87	......	75	...	71	69	......	......	......	......	......	63	......	......	...	...	...	...	...	...	...	87
	......	...	...	......	......	......	......	......	......	......	......	......	......	...	...	...	13	...	...	...	
88	......	76	...	72	70	......	......	......	......	......	64	......	......	...	...	...	13	...	...	...	88
	......	...	...	......	......	......	......	......	......	......	......	......	......	...	...	...	...	...	...	...	
89	......	...	...	......	......	......	......	......	......	......	......	......	......	...	...	...	...	...	...	...	89
	......	−72	73	− 76	65	......	......	......	......	......	......	......	− 62	...	...	...	14	...	...	...	
90	......	...	...	......	......	......	......	......	......	......	......	......	62	...	...	...	14	...	...	...	90
	......	−71	74	− 75	66	......	......	......	......	......	......	......	......	...	...	...	...	...	...	...	
91	......	...	71	......	67	......	......	......	......	......	......	......	......	...	...	...	...	...	...	...	91
	......	75	...	......	......	......	......	......	......	......	......	......	63	...	...	...	15	...	...	...	
92	......	...	72	......	68	......	......	......	......	......	......	......	......	...	...	...	15	...	...	...	92
	......	76	...	......	......	......	......	......	......	......	......	......	64	...	...	...	...	...	...	...	

TABLE II. (Continued.)

0	1	2	3	4	5	6	7	8	9	10	11	12	14	17	20	35	62	101	146	147	
93	....	73 ...	....	...... - 72	... 69	....	....	... ...	....	....	... ...	....	63 ...	....	... ...	... ...	... 16	....	... ...	... ...	93
94	....	74 ...	....	...... - 71	... 70	....	....	... ...	....	....	... ...	....	64 ...	....	... ...	... ...	16 ...	....	... ...	... ...	94
95	....	... ...	....	....	71 ...	....	....	... ...	....	....	... ...	....	... ...	...... - 62	... ...	... ...	... 17	....	... ...	... ...	95
96	....	... ...	....	....	72 ...	....	....	... ...	....	....	... ...	....	... ...	62	... ...	... ...	17 ...	....	... ...	... ...	96
97	....	... ...	....	....	... 73	....	....	... ...	....	....	... ...	....	... ...	 63	... ...	... ...	... 18	....	... ...	... ...	97
98	....	... ...	....	....	... 74	....	....	... ...	....	....	... ...	....	... ...	 64	... ...	... ...	18 ...	....	... ...	... ...	98
99	....	... ...	....	....	75 ...	....	....	... ...	....	....	... ...	....	... ...	63	... ...	... ...	... 19	....	... ...	... ...	99
100	....	... ...	....	....	76 ...	....	....	... ...	....	....	... ...	....	... ...	64	... ...	... ...	19 ...	....	... ...	... ...	100
101	116. -101	102 103	117. -102	118. -103	104 105	....	....	... ...	....	....	... ...	....	.. ...	....	... ...	... ...	... ...	 1	... ...	... ...	101
102	117. -103	... 101	...... -101	 116	... ...	....	....	... ...	....	....	... ...	....	... ...	....	... ...	... ...	... ...	3. - 2	... ...	... ...	102
103	118. -102	101 ...	 116	...... -101	... ...	....	....	... ...	....	....	... ...	....	... ...	....	... ...	... ...	... ...	2 4	... ...	... ...	103
104	119. -105	... ...	....	....	... 101	...... -101	 116	... ...	....	....	... ...	....	... ...	....	... ...	... ...	... ...	6. - 5	... ...	... ...	104
105	120. -104	... ...	....	....	101 ...	 116	...... -101	... ...	....	....	... ...	....	... ...	....	... ...	... ...	... ...	5 7	... ...	... ...	105
106	....	... 102	...... -103	 117	... ...	....	....	... 101	...... -101	 116	... ...	....	... ...	....	... ...	... ...	... ...	9. - 8	... ...	... ...	106
107	....	103 ...	 118	...... -102	... ...	....	....	101 ...	 116	...... -101	... ...	....	... ...	....	... ...	... ...	... ...	8 10	... ...	... ...	107
108	....	... 104	...... -105	 119	... 102	....	....	... ...	....	....	... 101	...... -101	... ...	....	... ...	... ...	... ...	12. - 11	... ...	... ...	108
109	....	105 ...	 120	...... -104	103 ...	....	....	... ...	....	....	101 ...	 116	... ...	....	... ...	... ...	... ...	11 13	... ...	... ...	109
110	....	... 105	...... -104	 120	102 ...	....	....	... ...	....	....	... ...	....	... 101	....	... ...	... ...	... ...	15. - 14	... ...	... ...	110
111	....	104 ...	 119	...... -105	... 103	....	....	... ...	....	....	... ...	....	101 ...	....	... ...	... ...	... ...	14 16	... ...	... ...	111
112	....	... ...	....	....	... 104	....	....	... ...	....	....	... ...	....	... ...	 101	... ...	... ...	... ...	18. - 17	... ...	... ...	112
113	....	... ...	....	....	105 ...	....	....	... ...	....	....	... ...	....	... ...	101	... ...	... ...	... ...	17 19	... ...	... ...	113
114	....	... ...	....	....	... ...	....	....	... ...	....	....	... ...	....	... ...	....	... ...	... ...	... 101	63. - 62	... ...	... ...	114
115	....	... ...	....	....	... ...	....	....	... ...	....	....	... ...	....	... ...	....	... ...	... ...	101 ...	62 64	... ...	... ...	115
116	101	117 118	103	102	119 120	....	....	... ...	....	....	... ...	....	... ...	....	... ...	... ...	... ...	1 131	... ...	... ...	116

Table II. (Continued.)

0	1	2	3	4	5	6	7	8	9	10	11	12	14	17	20	35	62	101	146	147	
117	102	 116	101	....	....	....	....	....	....	....	....	....	....	....	....	....	....	3	... ...	... ...	117
118	103	116	....	101	....	....	....	....	....	....	....	....	....	....	....	....	....	4	... ...	... ...	118
119	104	....	....	....	 116	101	....	....	....	....	....	....	....	....	....	....	....	6	... ...	... ...	119
120	105	....	....	....	116	....	101	....	....	....	....	....	....	....	....	....	....	7	... ...	... ...	120
121	....	 117	102	....	....	....	....	 116	101	....	....	....	....	....	....	....	....	9	... ...	... ...	121
122	....	118	....	103	....	....	....	116	....	101	....	....	....	....	....	....	....	10	... ...	... ...	122
123	....	 119	104	....	 117	....	....	....	....	....	 116	101	....	....	....	....	....	12	... ...	... ...	123
124	....	120	....	105	118	....	....	....	....	....	116	....	....	....	....	....	....	13	... ...	... ...	124
125	....	 120	105	....	117	....	....	....	....	....	....	....	 116	....	....	....	....	15	... ...	... ...	125
126	....	119	....	104	 118	....	....	....	....	....	....	....	116	....	....	....	....	16	... ...	... ...	126
127	....	....	....	....	 119	....	....	....	....	....	....	....	....	 116	....	....	....	18	... ...	... ...	127
128	....	....	....	....	120	....	....	....	....	....	....	....	....	116	....	....	....	19	... ...	... ...	128
129	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	 116	63	... ...	... ...	129
130	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	116	64	... ...	... ...	130
131	1	132 133	4	3	134 135	7	6	....	10	9	....	13	....	....	....	....	....	116	... ...	148 ...	131
132	3	 131	1	9	....	15	12	....	4	....	....	7	....	....	....	....	....	....	... ...	... ...	132
133	4	131	10	1	....	13	16	....	....	3	....	....	....	....	....	....	....	....	... ...	... ...	133
134	6	....	16	12	 131	1	18	....	....	....	....	4	....	....	....	....	....	....	... ...	... ...	134
135	7	....	13	15	131	19	1	....	....	....	....	....	....	....	....	....	....	....	... ...	... ...	135
136	9	 132	3	21	....	....	....	 131	1	....	....	15	....	....	....	....	....	....	... ...	... ...	136
137	10	133	....	4	....	....	....	131	....	1	....	....	....	....	....	.. ..	....	....	... ...	... ...	137
138	12	 134	6	....	 132	3	....	....	16	....	 131	1	....	....	....	....	....	....	... ...	... ...	138
139	13	135	....	7	133	....	4	....	....	15	131	....	....	....	....	....	....	....	... ...	... ...	139
140	15	 135	7	....	132	....	3	....	13	....	....	19	 131	....	....	....	....	....	... ...	... ...	140

TABLE II. (Continued.)

0	1	2	3	4	5	6	7	8	9	10	11	12	14	17	62	101	146	147	
141	16	134	....	6	 133	....	....	....	....	12	... ...	10 ...	131 ...	....	....	... ...	....	....	141
142	18	....	....	....	 134	6	....	....	....	....	... ...	16 ...	... ...	 131	....	... ...	....	....	142
143	19	....	....	....	135	....	7	....	....	....	... ...	... ...	... ...	131	....	... ...	....	....	143
144	....	....	....	....	....	....	....	....	....	....	... ...	... ...	... ...	....	 131	... ...	....	147	144
145	....	....	....	....	....	....	....	....	....	....	... ...	... ...	... ...	....	131	... ...	....	....	145
146	148. −147	150. −149	152. −151	154. −153	156. −155	....	....	....	....	....	... ...	... ...	... ...	....	...... −146	... ...	 62	− 63. 1	146
147	...... −146	151 153	149	...... −150	157 159	....	....	....	....	....	... ...	... ...	... ...	....	 148	... ...	63 1	....	147
148	146	152 154	150	...... −149	158 160	....	....	....	....	....	... ..	... ...	... ...	....	147	... ...	1 64	62 131	148
149	153. −152	...... −146	 147	...... −148	....	....	....	....	....	....	... ...	... ...	... ...	....	....	... ...	 2	...... − 3	149
150	154. −151	146	148.	...... −147	....	....	....	....	....	....	... ...	... ...	... ...	....	....	... ...	2	 4	150
151	...... −150	 147	..... −146	....	....	....	....	....	....	....	... ...	... ...	... ...	....	....	... ...	 3	...... − 2	151
152	...... −149	 148	146	....	....	....	....	....	....	....	... ...	... ...	... ...	....	....	... ...	3	....	152
153	149	147	....	...... −146	....	....	....	....	....	....	... ...	... ...	... ...	....	....	... ...	 4	2	153
154	150	148	....	146	....	....	....	....	....	....	... ...	... ...	... ...	....	....	... ...	4	....	154
155	159. −158	....	....	....	...... −146	 147	...... −148	....	....	....	... ...	... ...	... ...	....	....	... ...	 5	...... − 6	155
156	160. −157	....	....	....	146	 148	... .. −147	.. ..	....	....	... ...	... ...	... ...	....	....	... ...	5	 7	156
157	...... −156	....	....	....	 147	...... −146	....	....	....	....	... ...	... ...	... ...	....	....	... ...	 6	...... − 5	157
158	... .. −155	....	....	....	 148	146	....	....	....	....	... ...	... ...	... ...	....	....	... ...	6	....	158
159	155	....	....	....	147	....	...... −146	....	....	....	... ...	... ...	... ...	....	....	... ...	 7	5	159
160	156	....	....	....	148	....	146	....	....	....	... ...	... ...	... ...	....	....	... ...	7	....	160
161	....	149	 153	...... −152	....	....	....	...... −146	 147	...... −148	... ...	... ...	... ...	....	....	... ...	 8	...... − 9	161
162	....	150	 154	...... −151	....	....	....	146	 148	...... −147	... ...	... ...	... ...	....	....	... ...	8	 10	162
163	....	 151	...... −150	....	....	....	....	 147	 146	....	... ...	... ...	... ...	....	....	... ...	 9	...... − 8	163
164	....	 152	...... −149	....	....	....	....	147 148	146	....	... ...	... ...	... ...	....	....	... ...	9	....	164

TABLE II. (Continued.)

0	1	2	3	4	5	6	7	8	9	10	11	12	14	17	62	101	146	147	
165	...	153	......	149	......	......	......	......	......	......	......	......	......	......	...	......	...	8	165
	...	......	......	......	......	......	......	......	......	–146	......	......	......	......	...	......	10	......	
166	...	154	......	150	......	......	......	148	......	146	......	......	......	......	...	......	10	......	166
	...	......	......	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
167	...	155	......	......	149	......	......	......	......	......	......	......	......	......	...	......	...	......	167
	...	......	159	–158	......	......	......	......	......	......	–146	147	......	......	...	......	11	–12	
168	...	156	......	......	150	... ..	......	......	......	......	146	......	......	......	...	......	11	......	168
	...	......	160	–157	......	......	......	......	......	......	......	148	......	......	...	......	...	13	
169	...	......	......	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	169
	...	157	–156	......	151	......	......	......	......	......	147	–146	......	......	...	......	12	–11	
170	...	......	......	......	......	......	......	......	......	......	......	146	......	......	...	......	12	......	170
	...	158	–155	......	152	......	......	......	......	......	148	......	......	......	...	......	...	......	
171	...	159	......	......	153	......	......	......	......	......	147	......	......	......	...	......	...	11	171
	...	......	......	......	......	......	......	......	......	......	......	......	......	......	...	......	13	......	
172	...	160	......	156	154	......	....	......	......	......	148	......	......	......	...	......	13	......	172
	...	......	......	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
173	...	......	......	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	173
	...	–156	157	–160	149	......	......	......	......	......	......	......	–146	.. ...	...	......	14	–15	
174	...	......	......	......	......	......	......	......	......	......	......	......	146	......	...	......	14	......	174
	...	–155	158	–159	150	......	......	......	......	......	......	......	......	......	...	......	...	16	
175	...	......	155	......	151	......	......	......	......	......	......	......	......	......	...	......	...	......	175
	...	159	......	......	......	......	......	......	......	......	......	......	147	......	...	......	15	–14	
176	...	......	156	......	152	......	......	......	......	......	......	......	......	......	...	......	15	......	176
	...	160	......	......	.. ...	......	......	... ..	......	......	......	......	148	......	...	......	...	......	
177	...	157	......	......	......	......	......	......	......	......	......	......	147	......	...	......	...	14	177
	...	......	......	–156	153	......	......	......	......	......	......	......	......	......	...	......	16	......	
178	...	158	......	......	......	......	......	......	......	......	......	......	148	......	...	......	16	......	178
	...	......	......	–155	154	......	......	......	......	......	......	......	......	......	...	......	...	......	
179	...	......	... ..	......	155	......	......	......	......	......	......	......	......	......	...	......	...	......	179
	...	......	......	......	......	......	......	......	......	......	......	......	......	–146	...	......	17	–18	
180	...	......	......	......	156	......	......	......	......	......	......	......	......	146	...	......	17	......	180
	...	......	......	......	......	......	......	......	......	......	......	......	......	......	...	......	...	19	
181	...	......	......	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	181
	...	......	......	......	157	......	......	......	......	......	......	......	......	147	...	......	18	–17	
182	...	......	......	......	......	......	......	......	......	......	......	......	......	......	...	......	18	......	182
	...	......	......	......	158	......	......	......	......	......	......	......	......	148	...	......	...	......	
183	...	......	....	......	159	......	......	......	......	......	......	......	......	147	...	......	...	17	183
	...	......	......	......	......	......	......	......	......	......	......	......	......	......	...	......	19	......	
184	...	......	......	......	160	......	......	......	......	......	......	......	......	148	...	......	19	......	184
	...	......	......	......	......	......	......	......	......	......	......	......	......	......	...	......	...	......	
185	...	......	......	......	......	......	......	......	......	......	......	......	......	......	...	147.	...	......	185
	...	......	......	......	......	......	......	......	......	......	......	......	......	......	...	–146	...	......	
186	...	......	......	......	......	......	......	......	......	......	......	......	......	......	...	146	...	......	186
	...	......	......	......	......	......	......	......	......	......	......	......	......	......	...	148	...	......	

	38	59			38	59	
39	......	......	39	60	......	......	60
	1	......			......	1	
40	1	......	40	61	......	1	61
	......	......			......	......	

Table II. may be used in forming the developments required in the method employed by MM. Laplace and Damoiseau; for this purpose it is only necessary to make $t = \lambda' - \lambda_{,}$ instead of $n\,t - n_{,}t$

$$x = c\,\lambda' - \varpi \quad . \quad . \quad . \quad c\,n\,t - \varpi$$

$$z = c_{,}\lambda_{,} - \varpi_{,} \quad . \quad . \quad . \quad c_{,}n_{,}t - \varpi_{,}$$

$$\text{and } y = g\,\lambda' - \nu \quad . \quad . \quad . \quad g\,n\,t - \nu$$

The notation throughout is the same as that used Phil. Trans. 1830, p. 328, with the exception of the indices of the arguments.

In the elliptic movement;

$$a^5 r^{-5} = 1 + 5\,e^2\left(1 + \frac{21}{8}\,e^2\right) + 5\,e\left(1 + \frac{27}{8}\,e^2\right)\cos x + 10\,e^2\left(1 + \frac{31}{12}\,e^2\right)\cos 2\,x$$

$$+ \frac{145}{8}\,e^3 \cos 3\,x + \frac{745}{48}\,e^4 \cos 4\,x$$

$$a^4 r^{-4} = 1 + 3\,e^2 + 4\,e\cos x + 7\,e^2 \cos 2\,x$$

$$a^3 r^{-3} = 1 + \frac{3}{2}\,e^2\left(1 + \frac{5}{4}\,e^2\right) + 3e\left(1 + \frac{9}{8}\,e^2\right)\cos x + \frac{9}{2}\,e^2\left(1 + \frac{7}{9}\,e^2\right)\cos 2\,x$$

$$+ \frac{53}{8}\,e^3 \cos 3\,x + \frac{77}{8}\,e^4 \cos 4\,x$$

$$a^2 r^{-2} = 1 + \frac{e^2}{2}\left(1 + \frac{3}{4}\,e^2\right) + 2\,e\left(1 + \frac{3}{8}\,e^2\right)\cos x + \frac{5}{2}\,e^2\left(1 + \frac{2}{15}\,e^2\right)\cos 2\,x$$

$$+ \frac{13}{4}\,e^3 \cos 3\,x + \frac{103}{24}\,e^4 \cos 4\,x$$

$$a\,r^{-1} = 1 + e\left(1 - \frac{e^2}{8}\right)\cos x + e^2\left(1 - \frac{e^2}{3}\right)\cos 2\,x + \frac{9}{8}\,e^3 \cos 3\,x + \frac{4}{3}\,e^4 \cos 4\,x$$

$$\frac{r}{a} = 1 + \frac{e^2}{2} - e\left(1 - \frac{3\,e^2}{8}\right)\cos x - \frac{e^2}{2}\left(1 - \frac{2\,e^2}{3}\right)\cos 2\,x - \frac{3\,e^3}{8}\cos 3\,x - \frac{e^4}{3}\cos 4\,x$$

$$\frac{r^2}{a^2} = 1 + \frac{3\,e^2}{2} - 2\,e\left(1 - \frac{e^2}{8}\right)\cos x - \frac{e^2}{2}\left(1 - \frac{e^2}{3}\right)\cos 2\,x - \frac{e^3}{4}\cos 3\,x - \frac{e^4}{6}\cos 4\,x$$

$$\frac{r^3}{a^3} = 1 + 3\,e^2\left(1 + \frac{e^2}{8}\right) - 3\,e\left(1 + \frac{3}{8}\,e^2\right)\cos x - \frac{5}{8}\,e^4 \cos 2\,x + \frac{e^3}{8}\cos 3\,x + \frac{e^4}{8}\cos 4\,x$$

$$\frac{r^4}{a^4} = 1 + 5\,e^2 - 4\,e\cos x + e^2 \cos 2\,x$$

$$\begin{aligned}
\frac{a}{r} = {} & r_0 \\
& + r_1 \cos 2\,t \\
& + e\,r_2 \cos x \\
& + e\,r_3 \cos(2\,t - x) \\
& + e\,r_4 \cos(2\,t + x) \\
& + e_{,}\,r_5 \cos z \\
& + e_{,}\,r_6 \cos(2\,t - z) + \&c.\ \&c.
\end{aligned}$$

$$\begin{aligned}\lambda = {} & n t \\ & + \lambda_1 \cos 2 t \\ & + e \lambda_2 \cos x \\ & + e \lambda_3 \cos (2 t - x) \\ & + e \lambda_4 \cos (2 t + x) \\ & + e_{,} \lambda_5 \cos z \text{ \&c. \&c.}\end{aligned}$$

The quantities λ correspond to the quantities b in M. Damoiseau's notation.

$$\begin{aligned}s = {} & \gamma s_{146} \sin y \\ & + \gamma s_{147} \sin (2 t - y) \\ & + \gamma s_{148} \sin (2 t + y) \\ & + e \gamma s_{149} \sin (x - y) \text{ \&c. \&c.} \\ & \gamma = \tan \iota\end{aligned}$$

$$R = m_{,} \left\{ \frac{r^{\backprime} r_{,} \cos (\lambda - \lambda_{,})}{r_{,}^{3}} - \frac{1}{\{r^{2} - 2 r^{\backprime} r_{,} \cos (\lambda^{\backprime} - \lambda_{,}) + r_{,}^{2}\}^{\frac{1}{2}}} \right\}$$

$$= m_{,} \left\{ - \frac{1}{r_{,}} + \frac{r^{2}}{2 r_{,}^{3}} - \frac{3}{8} \frac{\{2 r^{\backprime} r_{,} \cos (\lambda^{\backprime} - \lambda_{,}) - r^{2}\}^{2}}{r_{,}^{5}} - \frac{15}{48} \frac{\{2 r^{\backprime} r_{,} \cos (\lambda^{\backprime} - \lambda_{,}) - r^{2}\}^{3}}{r_{,}^{7}} \right\}$$

$$= m_{,} \left\{ - \frac{1}{r_{,}} + \frac{r^{2}}{2 r_{,}^{3}} - \frac{3}{2} \frac{r^{\backprime 2} r_{,}^{2}}{r_{,}^{5}} \cos (\lambda^{\backprime} - \lambda_{,})^{2} + \frac{3}{2} \frac{r^{2} r^{\backprime} r_{,}}{r_{,}^{5}} \cos (\lambda - \lambda_{,}) - \frac{5}{2} \frac{r^{\backprime 3} r_{,}^{3}}{r_{,}^{7}} \cos (\lambda^{\backprime} - \lambda_{,})^{3} \right\}$$

$$\begin{aligned} = m_{,} \Bigg\{ & - \frac{1}{r_{,}} - \frac{r^{\backprime 2}}{4 r_{,}^{3}} \left\{ 1 + 3 \cos (2 \lambda^{\backprime} - 2 \lambda_{,}) - 2 s^{2} \right\} \\ & - \frac{r^{\backprime 3}}{8 r_{,}^{4}} \left\{ 3 (1 - 4 s^{2}) \cos (\lambda^{\backprime} - \lambda_{,}) + 5 \cos (3 \lambda^{\backprime} - 3 \lambda_{,}) \right\} \end{aligned}$$

$$r^{\backprime} r_{,} \genfrac{}{}{0pt}{}{\cos}{\sin} (\lambda^{\backprime} - \lambda_{,}) = r r_{,} \left\{ \cos^{2} \frac{\iota}{2} \genfrac{}{}{0pt}{}{\cos}{\sin} (\lambda - \lambda_{,}) + \sin^{2} \frac{\iota}{2} \genfrac{}{}{0pt}{}{\cos}{\sin} (\lambda + \lambda_{,} - 2 \nu) \right\}$$

$$\begin{aligned} =^{*} a a_{,} \cos^{2} \frac{\iota}{2} \Bigg\{ & \left(1 - \frac{e^{2}}{2} - \frac{e^{4}}{64}\right) \left(1 - \frac{e_{,}^{2}}{2} - \frac{e_{,}^{4}}{64}\right) \genfrac{}{}{0pt}{}{\cos}{\sin} t - \frac{3 e}{2} \left(1 - \frac{e_{,}^{2}}{2}\right) \genfrac{}{}{0pt}{}{\cos}{\sin} (t - x) \\ & + \frac{e}{2} \left(1 - \frac{3}{4} e^{2}\right) \left(1 - \frac{e_{,}^{2}}{2}\right) \genfrac{}{}{0pt}{}{\cos}{\sin} (t + x) + \frac{3}{8} e^{2} (1 - e^{2}) \left(1 - \frac{e_{,}^{2}}{2}\right) \genfrac{}{}{0pt}{}{\cos}{\sin} (t + 2 x) \\ & + \frac{e^{3}}{3} \genfrac{}{}{0pt}{}{\cos}{\sin} (t + 3 x) + \frac{125}{384} e^{4} \genfrac{}{}{0pt}{}{\cos}{\sin} (t + 4 x) + \frac{e^{2}}{8} \left(1 + \frac{e^{2}}{3}\right) \left(1 - \frac{e_{,}^{2}}{2}\right) \genfrac{}{}{0pt}{}{\cos}{\sin} (t - 2 x) \\ & + \frac{e^{3}}{24} \genfrac{}{}{0pt}{}{\cos}{\sin} (t - 3 x) + \frac{3}{128} e^{4} \genfrac{}{}{0pt}{}{\cos}{\sin} (t - 4 x) - \frac{3}{2} e_{,} \left(1 - \frac{e^{2}}{2}\right) \genfrac{}{}{0pt}{}{\cos}{\sin} (t + z) \\ & + \frac{9}{4} e e_{,} \genfrac{}{}{0pt}{}{\cos}{\sin} (t - x + z) - \frac{3}{4} e e_{,} \left(1 - \frac{3}{4} e^{2}\right) \genfrac{}{}{0pt}{}{\cos}{\sin} (t + x + z) \\ & - \frac{9}{16} e^{2} e_{,} \genfrac{}{}{0pt}{}{\cos}{\sin} (t + 2 x + z) - \frac{e^{3} e_{,}}{2} \genfrac{}{}{0pt}{}{\cos}{\sin} (t + 3 x + z) - \frac{3}{16} e^{2} e_{,} \genfrac{}{}{0pt}{}{\cos}{\sin} (t - 2 x + z) \end{aligned}$$

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[*] See Phil. Trans. 1830, p. 343.

$$-\frac{e^3 e_{\prime} \cos}{16 \sin}(t-3x+z)+\frac{e_{\prime}}{2}\left(1-\frac{3}{4}e_{\prime}^2\right)\left(1-\frac{e^2}{2}\right)\frac{\cos}{\sin}(t-z)$$

$$-\frac{3}{4}e e_{\prime}\left(1-\frac{3}{4}e_{\prime}^2\right)\frac{\cos}{\sin}(t-x-z)$$

$$+\frac{e e_{\prime}}{4}\left(1-\frac{3}{4}e^2\right)\left(1-\frac{3}{4}e_{\prime}^2\right)\frac{\cos}{\sin}(t+x-z)+\frac{3}{16}e^2 e_{\prime}\frac{\cos}{\sin}(t+2x-z)$$

$$+\frac{e^3 e_{\prime} \cos}{6 \sin}(t+3x-z)+\frac{e^2 e_{\prime} \cos}{16 \sin}(t-2x-z)+\frac{e^3 e_{\prime} \cos}{48 \sin}(t-3x-z)$$

$$+\frac{3}{8}e_{\prime}^2(1-e_{\prime}^2)\left(1-\frac{e^2}{2}\right)\frac{\cos}{\sin}(t-2z)-\frac{9}{16}e e_{\prime}^2\frac{\cos}{\sin}(t-x-2z)$$

$$+\frac{3}{16}e e_{\prime}^2\frac{\cos}{\sin}(t+x-2z)+\frac{9}{64}e^2 e_{\prime}^2\frac{\cos}{\sin}(t+2x-2z)$$

$$+\frac{3}{64}e^2 e_{\prime}^2\frac{\cos}{\sin}(t-2x-2z)+\frac{e_{\prime}^3 \cos}{3 \sin}(t-3z)-\frac{e e_{\prime}^3 \cos}{2 \sin}(t-x-3z)$$

$$+\frac{e e_{\prime}^3 \cos}{6 \sin}(t+x-3z)+\frac{125}{384}e_{\prime}^4\frac{\cos}{\sin}(t-4z)$$

$$+\frac{e_{\prime}^2}{8}\left(1+\frac{e_{\prime}^2}{3}\right)\left(1-\frac{e^2}{2}\right)\frac{\cos}{\sin}(t+2z)-\frac{3}{16}e e_{\prime}^2\frac{\cos}{\sin}(t-x+2z)$$

$$+\frac{e e_{\prime}^2 \cos}{16 \sin}(t+x+2z)+\frac{3}{64}e^2 e_{\prime}^2\frac{\cos}{\sin}(t+2x+2z)+\frac{e^2 e_{\prime}^2 \cos}{64 \sin}(t-2x+2z)$$

$$+\frac{e_{\prime}^3 \cos}{24 \sin}(t+3z)-\frac{e e_{\prime}^3 \cos}{16 \sin}(t-x+3z)+\frac{e e_{\prime}^3 \cos}{48 \sin}(t+x+3z)$$

$$+\frac{3}{128}e_{\prime}^4\frac{\cos}{\sin}(t+4z)\Big\}$$

$$+a a_{\prime}\sin^2\frac{\iota}{2}\Big\{\left(1-\frac{e^2+e_{\prime}^2}{2}\right)\frac{\cos}{\sin}(t-2y)-\frac{3}{2}e\frac{\cos}{\sin}(t+x-2y)+\frac{e \cos}{2 \sin}(t-x-2y)$$

$$+\frac{3}{8}e^2\frac{\cos}{\sin}(t-2x-2y)+\frac{e^2 \cos}{8 \sin}(t+2x-2y)-\frac{3}{2}e_{\prime}\frac{\cos}{\sin}(t+z-2y)$$

$$+\frac{9}{4}e e_{\prime}\frac{\cos}{\sin}(t+x+z-2y)-\frac{3}{4}e e_{\prime}\frac{\cos}{\sin}(t-x+z-2y)$$

$$+\frac{e_{\prime} \cos}{2 \sin}(t-z-2y)-\frac{3}{4}e e_{\prime}\frac{\cos}{\sin}(t+x-z-2y)+\frac{e e_{\prime} \cos}{4 \sin}(t-x-z-2y)$$

$r^{\prime 2} r_{\prime}^2 \cos(\lambda' - \lambda_{\prime})^2$

$$=a^2 a_{\prime}^2\cos^4\frac{\iota}{2}\Big\{\frac{1}{2}+\Big\{-\frac{1}{2}+\frac{9}{8}+\frac{1}{8}\Big\}(e^2+e_{\prime}^2)+\Big\{\frac{1}{2}-\frac{9}{8}-\frac{1}{8}-\frac{9}{8}+\frac{81}{32}+\frac{9}{32}$$

$$-\frac{1}{8}+\frac{9}{32}+\frac{1}{32}\Big\}e^2 e_{\prime}^2+\Big\{\frac{7}{64}-\frac{3}{16}+\frac{9}{128}+\frac{1}{128}\Big\}(e^4+e_{\prime}^4)$$

$$+\Biggl\{\frac{1}{2}+\Bigl\{-\frac{1}{2}-\frac{3}{4}\Bigr\}(e^2+e_{,}^2)+\Bigl\{\frac{1}{2}+\frac{3}{4}+\frac{3}{4}+\frac{9}{16}+\frac{9}{16}\Bigr\}e^2e_{,}^2$$

$$+\Bigl\{\frac{7}{64}+\frac{9}{16}+\frac{3}{64}\Bigr\}(e^4+e_{,}^4)\Biggr\}\cos 2\,t$$

[1]

$$+\Biggl\{-\frac{3}{2}+\frac{1}{2}+\Bigl\{\frac{3}{4}-\frac{5}{8}-\frac{3}{16}+\frac{3}{16}\Bigr\}e^2$$

$$+\Bigl\{\frac{3}{2}-\frac{1}{2}-\frac{27}{8}+\frac{9}{8}-\frac{3}{8}+\frac{1}{8}\Bigr\}e_{,}^2\Biggr\}e\cos x$$

[2] [5]*

$$+\Biggl\{-\frac{3}{2}+\Bigl\{\frac{3}{4}+\frac{1}{16}\Bigr\}e^2+\Bigl\{\frac{3}{2}+\frac{9}{8}+\frac{9}{8}\Bigr\}e_{,}^2\Biggr\}e\cos(2\,t-x)$$

[3] [7]

$$+\Biggl\{-\frac{1}{2}+\Bigl\{\frac{5}{8}-\frac{9}{16}\Bigr\}e^2+\Bigl\{-\frac{1}{2}-\frac{3}{8}-\frac{3}{8}\Bigr\}e_{,}^2\Biggr\}e\cos(2\,t+x)$$

[4] [6]

$$+\Biggl\{\frac{3}{8}+\frac{1}{8}-\frac{3}{4}+\Bigl\{-\frac{9}{16}-\frac{1}{48}+\frac{9}{16}-\frac{1}{16}+\frac{1}{6}\Bigr\}e^2$$

$$+\Bigl\{-\frac{3}{8}-\frac{1}{8}+\frac{3}{4}+\frac{27}{32}+\frac{9}{32}-\frac{27}{16}+\frac{3}{32}+\frac{1}{32}-\frac{3}{16}\Bigr\}e_{,}^2\Biggr\}e^2\cos 2\,x$$

[8] [17]

$$+\Biggl\{\frac{9}{8}+\frac{1}{8}+\Bigl\{-\frac{1}{48}+\frac{1}{48}\Bigr\}e^2$$

$$+\Bigl\{-\frac{9}{8}-\frac{1}{8}-\frac{3}{32}-\frac{27}{16}-\frac{3}{32}\Bigr\}e_{,}^2\Biggr\}e^2\cos(2\,t-2\,x)$$

[9] [19]

$$+\Biggl\{\frac{1}{8}+\frac{3}{8}+\Bigl\{-\frac{3}{16}-\frac{9}{16}-\frac{1}{2}\Bigr\}e^2$$

$$+\Bigl\{-\frac{1}{8}-\frac{3}{8}-\frac{9}{32}-\frac{3}{16}-\frac{9}{32}\Bigr\}e_{,}^2\Biggr\}e^2\cos(2\,t+2\,x)$$

[10] [18]

$$+\Biggl\{-\frac{3}{4}-\frac{3}{4}+\frac{9}{4}+\frac{1}{4}+\Bigl\{\frac{15}{16}+\frac{3}{8}-\frac{9}{8}-\frac{3}{32}-\frac{9}{32}\quad\frac{5}{16}$$

$$+\frac{3}{32}+\frac{9}{32}\Bigr\}(e^2+e_{,}^2)\Biggr\}e\,e_{,}\cos(x+z)$$

[11]

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* The coefficient of argument 5 being the same, e and $e_{,}$ changing places, that coefficient is not written down, in order to avoid useless repetition.

$$\begin{aligned}&+\Big\{-\frac{3}{4}-\frac{3}{4}+\Big\{\frac{3}{8}+\frac{3}{8}+\frac{1}{32}+\frac{1}{32}\Big\}e^2\\&\quad+\Big\{\frac{15}{16}+\frac{15}{16}+\frac{27}{32}+\frac{27}{32}\Big\}e_{\prime}^{2}\Big\}\,e\,e_{\prime}\cos(2t-x-z)\end{aligned}$$

[12] [13]

$$\begin{aligned}&+\Big\{\frac{9}{4}+\frac{1}{4}\quad\frac{3}{4}-\frac{3}{4}+\Big\{-\frac{9}{8}-\frac{5}{16}+\frac{9}{32}+\frac{3}{8}+\frac{15}{16}\\&\quad+\frac{3}{32}-\frac{9}{32}-\frac{3}{32}\Big\}(e^2+e_{\prime}^{2})\Big\}\,e\,e_{\prime}\cos(x-z)\end{aligned}$$

[14]

$$+\Big\{\frac{9}{4}+\frac{9}{4}+\Big\{-\frac{9}{8}-\frac{9}{8}-\frac{3}{32}-\frac{3}{32}\Big\}(e^2+e_{\prime}^{2})\Big\}\,e\,e_{\prime}\cos(2t-x+z)$$

[15]

$$+\Big\{\frac{1}{4}+\frac{1}{4}+\Big\{-\frac{5}{16}-\frac{9}{32}-\frac{5}{16}-\frac{9}{32}\Big\}(e_{\prime}+e_{\prime}^{2})\Big\}\cos(2t+x-z)$$

[16]

$$+\Big\{\frac{1}{3}+\frac{1}{24}-\frac{9}{16}+\frac{1}{16}\Big\}e\cos 3x$$

[20] [35]

$$+\Big\{\frac{1}{24}-\frac{3}{16}\Big\}e^3\cos(2t-3x)$$

[21] [37]

$$+\Big\{\frac{1}{3}+\frac{3}{16}\Big\}e^3\cos(2t+3x)$$

[22] [36]

$$+\Big\{-\frac{9}{16}+\frac{1}{16}+\frac{9}{8}-\frac{3}{8}+\frac{3}{16}-\frac{3}{16}\Big\}e^2e_{\prime}\cos(2x+z)$$

[23] [29]

$$+\Big\{\frac{1}{16}+\frac{9}{8}+\frac{1}{16}\Big\}e^2e_{\prime}\cos(2t-2x-z)$$

[24] [31]

$$+\Big\{-\frac{9}{16}-\frac{3}{8}-\frac{9}{16}\Big\}e^2e_{\prime}\cos(2t+2x+z)$$

[25] [30]

$$+\Big\{-\frac{3}{16}+\frac{3}{16}-\frac{3}{8}+\frac{9}{8}-\frac{9}{16}+\frac{1}{16}\Big\}e^2e_{\prime}\cos(2x-z)$$

[26] [32]

$$+\Big\{-\frac{3}{16}-\frac{27}{8}-\frac{3}{16}\Big\}e^2e_{\prime}\cos(2t-2x+z)$$

[27] [33]

$$+\Big\{\frac{3}{16}+\frac{1}{8}+\frac{3}{16}\Big\}e^2e_{\prime}\cos(2t+2x-z)$$

[28] [34]

$$+\Big\{\frac{125}{384}+\frac{3}{128}-\frac{1}{2}+\frac{1}{48}+\frac{3}{64}\Big\}e^4\cos 4x$$

[38] [59]

$$+\left\{\frac{1}{128}+\frac{3}{128}-\frac{1}{16}\right\}e^4\cos(2t-4x)$$

[39] [61]

$$+\left\{\frac{9}{128}+\frac{125}{384}+\frac{1}{6}\right\}e^4\cos(2t+4x)$$

[40] [60]

$$+\left\{-\frac{1}{2}+\frac{1}{48}+\frac{27}{32}+\frac{1}{32}-\frac{9}{32}+\frac{1}{6}-\frac{3}{32}-\frac{1}{16}\right\}e^3\cos(3x+z)$$

[41] [53]

$$+\left\{\frac{1}{48}-\frac{3}{32}-\frac{3}{32}+\frac{1}{48}\right\}e^3e_{,}\cos(2t-3x-z)$$

[42] [55]

$$+\left\{-\frac{1}{2}-\frac{9}{32}-\frac{9}{32}-\frac{1}{2}\right\}e^3e_{,}\cos(2t+3x+z)$$

[43] [54]

$$+\left\{-\frac{1}{16}+\frac{1}{6}-\frac{9}{32}-\frac{3}{32}+\frac{27}{32}-\frac{1}{2}+\frac{1}{32}+\frac{1}{48}\right\}e^3e_{,}\cos(3x-z)$$

[44] [56]

$$+\left\{-\frac{1}{16}+\frac{9}{32}+\frac{9}{32}-\frac{1}{16}\right\}e^3e_{,}\cos(2t-3x+z)$$

[45] [57]

$$+\left\{\frac{1}{6}+\frac{3}{32}+\frac{3}{32}+\frac{1}{6}\right\}e^2e_{,}\cos(2t+3x-z)$$

[46] [58]

$$+\left\{\frac{3}{64}+\frac{3}{64}-\frac{3}{32}-\frac{9}{32}+\frac{9}{64}+\frac{1}{64}-\frac{3}{32}+\frac{9}{16}-\frac{9}{32}\right\}e^2e_{,}^2\cos(2x+2z)$$

[47]

$$+\left\{\frac{9}{32}+\frac{3}{64}+\frac{27}{32}+\frac{3}{64}+\frac{1}{32}\right\}e^2e_{,}^2\cos(2t-2x+2z)$$

[48]

$$+\left\{\frac{9}{32}+\frac{3}{64}+\frac{1}{32}+\frac{3}{64}+\frac{27}{32}\right\}e^2e_{,}^2\cos(2t+2x+2z)$$

[49]

$$+\left\{\frac{9}{64}+\frac{1}{64}-\frac{9}{32}-\frac{3}{32}+\frac{3}{64}+\frac{3}{64}-\frac{9}{32}+\frac{9}{16}-\frac{3}{32}\right\}e^2e_{,}^2\cos(2x-2z)$$

[50]

$$+\left\{\frac{81}{32}+\frac{1}{64}+\frac{9}{32}+\frac{1}{64}+\frac{9}{32}\right\}e^2e_{,}^2\cos(2t-2x+2z)$$

[51]

$$+\left\{\frac{1}{32}+\frac{9}{64}+\frac{3}{32}+\frac{9}{64}+\frac{3}{32}\right\}e^2e_{,}^2\cos(2t+2x-2z)$$

[52]

$$+\,a^2a_{,}^2\sin^2\frac{\iota}{2}\cos^2\frac{\iota}{2}\left[\left\{1+\left\{-1-\frac{3}{4}-\frac{3}{4}\right\}e^2+\left\{-1+\frac{9}{4}+\frac{1}{4}\right\}e_{,}^2\right\}\cos 2y\right.$$

[62]

$$\left.+\left\{1+\left\{-1+\frac{9}{4}+\frac{1}{4}\right\}e^2+\left\{-1-\frac{3}{4}-\frac{3}{4}\right\}e_{,}^2\right\}\cos(2t-2y)\right.$$

[63]

$$+\left\{-\frac{3}{2}-\frac{3}{2}\right\}\underset{[65]}{e\cos(x-2y)}+\left\{\frac{1}{2}+\frac{1}{2}\right\}\underset{[66]}{e\cos(x+2y)}$$

$$+\left\{\frac{1}{2}-\frac{3}{2}\right\}\underset{[67]}{e\cos(2t-x-2y)}$$

$$+\left\{-\frac{3}{2}+\frac{1}{2}\right\}\underset{[69]}{e\cos(2t+x-2y)}+\left\{-\frac{3}{2}+\frac{1}{2}\right\}\underset{[71]}{e_{,}\cos(z-2y)}$$

$$+\left\{\frac{1}{2}-\frac{3}{2}\right\}\underset{[72]}{e_{,}\cos(z+2y)}+\left\{\frac{1}{2}+\frac{1}{2}\right\}\underset{[73]}{e_{,}\cos(2t-z-2y)}$$

$$+\left\{-\frac{3}{2}-\frac{3}{2}\right\}\underset{[75]}{e_{,}\cos(2t+z-2y)}+\left\{\frac{1}{8}+\frac{9}{4}+\frac{1}{8}\right\}\underset{[77]}{e^{2}\cos(2x-2\ \ )}$$

$$+\left\{\frac{3}{8}+\frac{1}{4}+\frac{3}{8}\right\}\underset{[78]}{e^{2}\cos(2x+2y)}$$

$$+\left\{\frac{3}{8}-\frac{3}{4}+\frac{1}{8}\right\}\underset{[79]}{e^{2}\cos(2t-2x-2y)}+\left\{\frac{1}{8}-\frac{3}{4}+\frac{3}{8}\right\}\underset{[81]}{e^{2}\cos(2t+2x-2y)}$$

$$+\left\{\frac{9}{4}+\frac{9}{4}-\frac{3}{4}-\frac{3}{4}\right\}\underset{[83]}{e\,e_{,}\cos(x+z-2y)}$$

$$+\left\{\frac{1}{4}+\frac{1}{4}-\frac{3}{4}-\frac{3}{4}\right\}\underset{[84]}{e\,e_{,}\cos(x+z+2y)}$$

$$+\left\{\frac{1}{4}-\frac{3}{4}+\frac{1}{4}-\frac{3}{4}\right\}\underset{[85]}{e\,e_{,}\cos(2t-x-z-2y)}$$

$$+\left\{\frac{9}{4}-\frac{3}{4}+\frac{9}{4}-\frac{3}{4}\right\}\underset{[87]}{e\,e_{,}\cos(2t+x+z+2y)}$$

$$+\left\{-\frac{3}{4}-\frac{3}{4}+\frac{9}{4}+\frac{9}{4}\right\}\underset{[89]}{e\,e_{,}\cos(x-z-2y)}$$

$$+\left\{-\frac{3}{4}-\frac{3}{4}+\frac{1}{4}+\frac{1}{4}\right\}\underset{[90]}{e\,e_{,}\cos(x-z+2y)}$$

$$+\left\{-\frac{3}{4}+\frac{9}{4}-\frac{3}{4}+\frac{9}{4}\right\}\underset{[91]}{e\,e_{,}\cos(2t-x+z-2y}$$

$$+\left\{-\frac{3}{4}+\frac{1}{4}-\frac{3}{4}+\frac{1}{4}\right\}\underset{[93]}{e\,e_{,}\cos(2t+x-z-2y}$$

$$+\left\{-\frac{3}{4}+\frac{3}{8}\right\}e_{,}^{2}\underset{[95]}{\cos(2z-2y)}+\left\{-\frac{3}{4}+\frac{1}{8}\right\}e_{,}^{2}\underset{[96]}{\cos(2z+2y)}$$

$$+\left\{\frac{1}{4}+\frac{3}{8}\right\}e_{,}^{2}\underset{[97]}{\cos(2t-2z-2y)}+\left\{\frac{9}{4}+\frac{1}{8}\right\}e_{,}^{2}\underset{[99]}{\cos(2t+2z-2y)}$$

$$+a^{2}a_{,}^{2}\sin^{4}\frac{\iota}{2}\left\{\frac{1}{2}+\frac{1}{2}\underset{[63]}{\cos(2t-2y)}\right\}$$

$$r^{\prime 2}r^{2}\cos(\lambda^{\prime}-\lambda)^{2}$$

$$=a^{2}a_{,}^{2}\cos^{4}\frac{\iota}{2}\left\{\frac{1}{2}+\frac{3}{4}(e^{2}+e_{,}^{2})+\frac{9}{8}e^{2}e_{,}^{2}+\left\{\frac{1}{2}-\frac{5}{4}(e^{2}+e_{,}^{2})\right.\right.$$

$$\left.+\frac{23}{32}(e^{4}+e_{,}^{4})+\frac{25}{8}e^{2}e_{,}^{2}\right\}\underset{[1]}{\cos 2t}+\left\{-1+\frac{e^{2}}{8}-\frac{3}{2}e_{,}^{2}\right\}\underset{[2]\ [5]}{e\cos x}$$

$$+\left\{-\frac{3}{2}+\frac{13}{16}e^{2}+\frac{15}{4}e_{,}^{2}\right\}\underset{[3]\ [7]}{e\cos(2t-x)}+\left\{\frac{1}{2}-\frac{19}{16}e^{2}-\frac{5}{4}e_{,}^{2}\right\}\underset{[4]\ [6]}{e\cos(2t+x)}$$

$$+\left\{-\frac{1}{4}+\frac{1}{12}e^{2}-\frac{3}{8}e_{,}^{2}\right\}\underset{[8]\ [17]}{e^{2}\cos 2x}$$

$$+\left\{\frac{5}{4}-\frac{25}{8}e_{,}^{2}\right\}\underset{[9]\ [19]}{e^{2}\cos(2t-2x)}+\left\{\frac{1}{2}-\frac{5}{4}e^{2}-\frac{5}{4}e_{,}^{2}\right\}\underset{[10]\ [18]}{e\cos(2t+2x)}$$

$$+\left\{1-\frac{1}{8}(e^{2}+e_{,}^{2})\right\}\underset{[11]}{e\,e_{,}\cos(x+z)}$$

$$+\left\{-\frac{3}{2}+\frac{13}{16}e^{2}+\frac{57}{16}e_{,}^{2}\right\}\underset{[12]\ [13]}{e\,e_{,}\cos(2t-x-z)}$$

$$+\left\{1-\frac{(e^{2}+e_{,}^{2})}{8}\right\}\underset{[14]}{e\,e_{,}\cos(x-z)}$$

$$+\left\{\frac{9}{2}-\frac{39}{16}(e^{2}+e_{,}^{2})\right\}\underset{[15]}{e\,e_{,}\cos(2t-x+z)}$$

$$+\left\{\frac{1}{2}-\frac{19}{16}(e^{2}+e_{,}^{2})\right\}\underset{[16}{e\,e_{,}\cos(2t+x-z)}-\underset{[20]\ [35]}{\frac{e^{3}}{8}\cos 3x}$$

$$-\underset{[21]\ [37]}{\frac{7e^{3}}{48}\cos(2t-3x)}+\underset{[22]\ [36]}{\frac{25e^{3}}{48}\cos(2t+3x)}+\underset{[23]\ [29]}{\frac{e^{2}e_{,}}{4}\cos(2x+z)}$$

$$+\underset{[24]\,[31]}{\frac{5}{4}e^{2}e_{,}\cos(2t-2x-z)}-\underset{[25]\,[30]}{\frac{25}{16}e^{2}e_{,}\cos(2t+2x+z)}+\underset{[26]\,[32]}{\frac{e^{2}e_{,}}{4}\cos(2x-z)}$$

$$-\underset{[27]\,[33]}{\frac{15}{4}e^{2}e_{,}\cos(2t-2x+z)}+\underset{[28]\,[34]}{\frac{e^{2}e_{,}}{2}\cos(2t+2x-z)}$$

$$-\underset{[38]\,[59]}{\frac{e^{4}}{12}\cos 4x}-\underset{[39]\,[61]}{\frac{e^{4}}{32}\cos(2t-4x)}+\underset{[40]\,[60]}{\frac{9}{16}e^{4}\cos(2t+4x)}+\underset{[41]\,[53]}{\frac{e^{3}e_{,}}{8}\cos(3x+z)}$$

$$-\underset{[42]\,[55]}{\frac{7}{48}e^{3}e_{,}\cos(2t-3x-z)}-\underset{[43]\,[54]}{\frac{25}{16}e^{2}e_{,}\cos(2t+3x+z)}$$

$$+\underset{[44]\,[56]}{\frac{e^{3}e_{,}}{8}\cos(3x-z)}+\underset{[45]\,[57]}{\frac{7}{16}e^{3}e_{,}\cos(2t-3x+z)}+\underset{[46]\,[58]}{\frac{25}{48}e^{3}e_{,}\cos(2t+3x-z)}$$

$$+\underset{[47]}{\frac{e^{2}e_{,}}{16}\cos(2x+2z)}+\underset{[48]}{\frac{5}{4}e^{2}e_{,}^{2}\cos(2t-2x-2z)}$$

$$+\underset{[49]}{\frac{5}{4}e^{2}e_{,}\cos(2t+2x+2z)}+\underset{[50]}{\frac{e^{2}e_{,}^{2}}{16}\cos(2x-2z)}$$

$$+\underset{[51]}{\frac{25}{8}e^{2}e_{,}^{2}\cos(2t-2x+2z)}+\underset{[52]}{\frac{e^{2}e_{,}^{2}}{2}\cos(2t+2x-2z)}$$

$$+a^{2}a_{,}^{2}\cos^{2}\frac{\iota}{2}\sin^{2}\frac{\iota}{2}\left\{\underset{[62]}{\left\{1-\frac{5}{2}e^{2}+\frac{3}{2}e_{,}^{2}\right\}\cos 2y}+\underset{[63]}{\left\{1+\frac{3}{2}e^{2}-\frac{5}{2}e_{,}^{2}\right\}\cos(2t-2y)}\right.$$

$$-\underset{[65]}{3e\cos(x-2y)}+\underset{[66]}{e\cos(x+2y)}-\underset{[67]}{e\cos(2t-x-2y)}$$

$$-\underset{[69]}{e\cos(2t+x-2y)}-\underset{[71]}{e_{,}\cos(z-2y)}-\underset{[72]}{e_{,}\cos(z+2y)}$$

$$+\underset{[73]}{e_{,}\cos(2t-z-2y)}-\underset{[75]}{3e_{,}\cos(2t+z-2y)}$$

$$+\underset{[77]}{\frac{5}{2}e^{2}\cos(2x-2y)}+\underset{[78]}{e^{2}\cos(2x+2y)}$$

$$-\underset{[79]}{\frac{e^{2}}{4}\cos(2t-2x-2y)}-\underset{[81]}{\frac{e^{2}}{4}\cos(2t+2x-2y)}$$

$$+ \underset{[83]}{3\,e\,e_{,} \cos (x + z - 2\,y)} - \underset{[84]}{e\,e_{,} \cos (x + z + 2\,y)}$$

$$- \underset{[85]}{e\,e_{,} \cos (2\,t - x - z - 2\,y)} + \underset{[87]}{3\,e\,e_{,} \cos (2\,t + x + z - 2\,y)}$$

$$+ \underset{[89]}{3\,e\,e_{,} \cos (x - z - 2\,y)} - \underset{[90]}{e\,e_{,} \cos (x - z + 2\,y)}$$

$$+ \underset{[91]}{3\,e\,e_{,} \cos (2\,t - x + z - 2\,y)} - \underset{[93]}{e\,e_{,} \cos (2\,t + x - z - 2\,y)}$$

$$- \underset{[95]}{\frac{3}{8}\,e_{,}^{2} \cos (2\,z - 2\,y)} - \underset{[96]}{\frac{5}{8}\,e_{,}^{2} \cos (2\,z + 2\,y)}$$

$$+ \underset{[97]}{\frac{5\,e_{,}^{2}}{8} \cos (2\,t - 2\,z - 2\,y)} + \underset{[99]}{\frac{19}{8}\,e_{,}^{2} \cos (2\,t + 2\,z - 2\,y)} \Big\}$$

$$+ a^{2}\,a_{,}^{2} \sin^{4} \frac{\iota}{2} \left\{ \frac{1}{2} + \underset{[63]}{\frac{1}{2} \cos (2\,t - 2\,y)} \right\}$$

$$\frac{r^{2}}{2\,r_{,}^{3}} = \frac{a^{2}}{a_{,}^{3}} \left\{ \frac{1}{2} + \frac{3}{4}\,e^{2} + \frac{3}{4}\,e_{,}^{2} + \frac{9}{8}\,e^{2}\,e_{,}^{2} + \frac{15}{16}\,e_{,}^{4} - \underset{[2]}{e \left\{ 1 - \frac{e^{2}}{8} + \frac{3}{2}\,e_{,}^{2} \right\} \cos x} \right.$$

$$+ \underset{[5]}{\frac{3}{2}\,e_{,} \left\{ 1 + \frac{3}{2}\,e^{2} + \frac{9}{8}\,e_{,}^{2} \right\} \cos z} - \underset{[8]}{\frac{e^{2}}{4} \left\{ 1 - \frac{e^{2}}{3} + \frac{3\,e_{,}^{2}}{2} \right\} \cos 2\,x}$$

$$- \underset{[11]}{\frac{3}{2}\,e\,e_{,} \left\{ 1 - \frac{e^{2}}{8} + \frac{9}{8}\,e_{,}^{2} \right\} \cos (x + z)}$$

$$- \underset{[14]}{\frac{3}{2}\,e\,e_{,} \left\{ 1 - \frac{e^{2}}{8} + \frac{9}{8}\,e_{,}^{2} \right\} \cos (x - z)}$$

$$+ \underset{[17]}{\frac{9}{4}\,e_{,}^{2} \left\{ 1 + \frac{7}{9}\,e_{,}^{2} + \frac{3}{2}\,e^{2} \right\} \cos 2\,z} - \underset{[20]}{\frac{e^{3}}{8} \cos 3\,x} - \underset{[23]}{\frac{3}{8}\,e^{2}\,e_{,} \cos (2\,x + z)}$$

$$- \underset{[26]}{\frac{3}{8}\,e^{2}\,e_{,} \cos (2\,x - z)} - \underset{[29]}{\frac{9}{4}\,e\,e_{,}^{2} \cos (x + 2\,z)} - \underset{[32]}{\frac{9}{4}\,e\,e_{,}^{2} \cos (x - 2\,z)}$$

$$+ \underset{[35]}{\frac{53}{16}\,e_{,}^{3} \cos 3\,z} - \underset{[38]}{\frac{e^{4}}{12} \cos 4\,x} - \underset{[41]}{\frac{3\,e^{3}\,e_{,}}{16} \cos (3\,x + z)}$$

$$-\frac{3}{16}e^3 e_{\prime}\cos(3x-z)-\frac{9}{16}e^2 e_{\prime}^2\cos(2x+2z)-\frac{9}{16}e^2 e_{\prime}\cos(2x-2z)$$

[44] [47] [50]

$$-\frac{53}{16}e\,e_{\prime}^3\cos(x+3z)-\frac{53}{16}e\,e_{\prime}^3\cos(x-3z)+\frac{77}{16}e_{\prime}^4\cos 4z$$

[53] [56] [59]

Terms in R multiplied by $-\frac{3}{2}\cos^4\frac{\iota}{2}\frac{a^2}{a_{\prime}^3}$

$$=^{*}\left\{\frac{1}{2}+\frac{3}{4}(e^2+e_{\prime}^2)+\frac{9}{8}e^2 e_{\prime}^2\right\}\left\{1+5e_{\prime}^2+\frac{105}{8}e_{\prime}^4\right\}$$

[0]

$$+\left\{-1+\frac{e_{\prime}^2}{8}-\frac{3}{2}e\right\}\left\{\frac{5}{2}e_{\prime}^2+\frac{135}{16}e_{\prime}^4\right\}-\frac{5}{4}e_{\prime}^4$$

$$+\Bigg\{\left\{\frac{1}{2}-\frac{5}{4}(e^2+e_{\prime}^2)+\frac{23}{32}(e^4+e_{\prime}^4)+\frac{25}{8}e^2 e_{\prime}^2\right\}\left\{1+5e_{\prime}^2+\frac{105}{8}e_{\prime}^4\right\}$$

$$+\left\{\frac{1}{2}-\frac{19}{16}e_{\prime}^2-\frac{5}{4}e^2-\frac{3}{2}+\frac{13}{16}e_{\prime}^2+\frac{15}{4}e^2\right\}\left\{\frac{5}{2}e_{\prime}^2+\frac{135}{16}e_{\prime}^4\right\}$$

$$+\left\{\frac{5}{2}+\frac{25}{4}\right\}e_{\prime}^4\Bigg\}\cos 2t$$

[1]

$$+\left\{-1+\frac{e^2}{8}-\frac{3}{2}e_{\prime}^2-5e_{\prime}^2+\frac{5}{2}e_{\prime}^2+\frac{5}{2}e_{\prime}^2\right\}e\cos x$$

[2]

$$+\left\{-\frac{3}{2}+\frac{13}{16}e^2+\frac{15}{4}e_{\prime}^2-\frac{15}{2}e_{\prime}^2-\frac{15}{4}e_{\prime}^2+\frac{45}{4}e_{\prime}^2\right\}e\cos(2t-x)$$

[3]

$$+\left\{\frac{1}{2}-\frac{19}{16}e^2-\frac{5}{4}e_{\prime}^2+\frac{5}{2}e_{\prime}^2+\frac{5}{4}e_{\prime}^2-\frac{15}{4}e_{\prime}^2\right\}e\cos(2t+x)$$

[4]

$$+\left\{-1+\frac{e_{\prime}^2}{8}-\frac{3}{2}e^2-5e_{\prime}^2+\frac{5}{2}+\frac{15}{4}e^2+\frac{15}{4}e_{\prime}^2+\frac{135}{16}e_{\prime}^2-\frac{5}{8}e_{\prime}^2-5e_{\prime}^2\right\}e_{\prime}\cos z$$

[5]

$$+\left\{\frac{1}{2}-\frac{19}{16}e_{\prime}^2-\frac{5}{4}e^2+\frac{5}{2}e_{\prime}^2+\frac{5}{4}e_{\prime}^2+\frac{5}{4}-\frac{25}{8}e^2-\frac{25}{8}e_{\prime}^2\right.$$

$$\left.+\frac{135}{32}e_{\prime}^2-\frac{15}{2}e_{\prime}^2\right\}e_{\prime}\cos(2t-z)$$

[6]

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* This multiplication of $r^2 r_{\prime}^2\cos(\lambda'-\lambda_{\prime})^2$ by $r_{\prime}^5$ may be effected at once by means of Table II.

$$+\left\{-\frac{3}{2}+\frac{13}{16}e_{,}^2+\frac{15}{4}e^2-\frac{15}{2}e_{,}^2+\frac{5}{4}-\frac{25}{8}e^2-\frac{25}{8}e_{,}^2+\frac{135}{32}e_{,}^2+\frac{25}{8}e_{,}^2+\frac{5}{2}e_{,}^2\right\}e_{,}\cos(2t+z)$$

[7]

$$+\left\{-\frac{1}{4}+\frac{e^2}{12}-\frac{3}{8}e_{,}^2-\frac{5}{4}e_{,}^2+\frac{5}{8}e_{,}^2+\frac{5}{8}e_{,}^2\right\}e^2\cos 2x$$

[8]

$$+\left\{\frac{5}{4}-\frac{25}{8}e_{,}^2+\frac{25}{4}e_{,}^2+\frac{25}{8}e_{,}^2-\frac{75}{8}e_{,}^2\right\}e^2\cos(2t-2x)$$

[9]

$$+\left\{\frac{1}{2}-\frac{5}{4}e^2-\frac{5}{4}e_{,}^2+\frac{5}{2}e_{,}^2+\frac{5}{4}e_{,}^2-\frac{15}{4}e_{,}^2\right\}e^2\cos(2t+2x)$$

[10]

$$+\left\{1-\frac{e^2}{8}-\frac{e_{,}^2}{8}+5e_{,}^2-\frac{5}{2}+\frac{5e^2}{16}-\frac{15}{4}e_{,}^2-\frac{135}{16}e_{,}^2+\frac{5e_{,}^2}{8}+5e_{,}^2\right\}ee_{,}\cos(x+z)$$

[11]

$$+\left\{-\frac{3}{2}+\frac{13}{16}e^2+\frac{57}{16}e_{,}^2-\frac{15}{2}e_{,}^2-\frac{15}{4}e_{,}^2-\frac{15}{4}+\frac{65}{32}e^2+\frac{75}{8}e_{,}^2\right.$$

$$\left.-\frac{405}{32}e_{,}^2+\frac{45}{2}e_{,}^2\right\}ee_{,}\cos(2t-x-z)$$

[12]

$$+\left\{-\frac{3}{2}+\frac{13}{16}e_{,}^2+\frac{57}{16}e^2-\frac{15}{2}e_{,}^2+\frac{5}{4}\right.$$

$$\left.-\frac{95}{32}e^2-\frac{25}{8}e_{,}^2+\frac{135}{32}e_{,}^2+\frac{25}{8}e_{,}^2+\frac{5}{2}e_{,}^2\right\}ee_{,}\cos(2t+x+z)$$

[13]

$$+\left\{1-\frac{e^2}{8}-\frac{e_{,}^2}{8}+5e_{,}^2+\frac{5e_{,}^2}{8}-\frac{5}{2}+\frac{5}{16}e^2-\frac{15}{4}e_{,}^2-\frac{135}{16}e_{,}^2+5e_{,}^2\right\}ee_{,}\cos(x-z)$$

[14]

$$+\left\{\frac{9}{2}-\frac{39}{16}e^2-\frac{39}{16}e_{,}^2+\frac{45}{2}e_{,}^2-\frac{15}{4}+\frac{65}{32}e^2+\frac{75}{8}e_{,}^2-\frac{405}{32}e_{,}^2\right.$$

$$\left.-\frac{75}{8}e_{,}^2-\frac{15}{2}e_{,}^2\right\}ee_{,}\cos(2t-x+z)$$

[15]

$$+\left\{\frac{1}{2}-\frac{19}{16}e^2-\frac{19}{16}e_{,}^2+\frac{5}{2}e_{,}^2+\frac{5}{4}e_{,}^2+\frac{5}{4}-\frac{95}{32}e^2-\frac{25}{8}e_{,}^2+\frac{135}{32}e_{,}^2-\frac{15}{2}e_{,}^2\right\}ee_{,}\cos(2t+x-z)$$

[16]

$$+\left\{-\frac{1}{4}+\frac{e_{,}^2}{12}-\frac{3}{8}e^2-\frac{5}{4}e_{,}^2-\frac{5}{2}+\frac{5}{16}e_{,}^2-\frac{15}{4}e^2-\frac{135}{16}e_{,}^2-\frac{5}{16}e_{,}^2+5\right.$$

$$\left.+\frac{15}{2}e^2+\frac{15}{2}e_{,}^2+\frac{155}{12}e_{,}^2-\frac{145}{16}e_{,}^2\right\}e_{,}^2\cos 2z$$

[17]

$$+\left\{\frac{1}{2}-\frac{5}{4}e_{,}^2-\frac{5}{4}e^2+\frac{5}{2}e_{,}^2+\frac{125}{96}e_{,}^2+\frac{5}{4}-\frac{95}{32}e_{,}^2-\frac{25}{8}e^2+\frac{135}{32}e_{,}^2+\frac{5}{2}-\frac{25}{4}e^2-\frac{25}{4}e_{,}^2\right.$$

$$\left.+\frac{155}{24}e_{,}^2-\frac{435}{32}e_{,}^2\right\}e_{,}^2\cos(2t-2z)$$

[18]

$$+\left\{\frac{5}{4}-\frac{25}{8}e^2+\frac{25}{4}e_{,}^2-\frac{15}{4}+\frac{65}{32}e_{,}^2+\frac{75}{8}e^2-\frac{405}{32}e_{,}^2-\frac{35}{96}e_{,}^2+\frac{5}{2}-\frac{25}{4}e^2-\frac{25}{4}e_{,}^2\right.$$

$$\underset{[19]}{\left.+\frac{155}{24}e_{,}^2+\frac{145}{32}e_{,}^2\right\}e_{,}^2\cos(2t+2z)}$$

$$-\underset{[20]}{\frac{e^3}{8}\cos 3x}-\underset{[21]}{\frac{7}{48}e^3\cos(2t-3x)}+\underset{[22]}{\frac{25}{48}e^3\cos(2t+3x)}+\underset{[23]}{\left\{\frac{1}{4}-\frac{5}{8}\right\}e^2e_{,}\cos(2x+z)}$$

$$+\underset{[24]}{\left\{\frac{5}{4}+\frac{25}{8}\right\}e^2e_{,}\cos(2t-2x-z)}-\underset{[25]}{\left\{-\frac{3}{2}+\frac{5}{4}\right\}e^2e_{,}\cos(2t+2x+z)}$$

$$+\underset{[26]}{\left\{\frac{1}{4}\quad\frac{5}{8}\right\}e^2e_{,}\cos(2x-z)}+\underset{[27]}{\left\{-\frac{15}{4}+\frac{25}{8}\right\}e^2e_{,}\cos(2t-2x+z)}$$

$$+\underset{[28]}{\left\{\frac{1}{2}+\frac{5}{4}\right\}e^2e_{,}\cos(2t+2x-z)}+\underset{[29]}{\left\{\frac{1}{4}+\frac{5}{2}-5\right\}e\,e_{,}^2\cos(x+2z)}$$

$$+\underset{[30]}{\left\{-\frac{3}{2}-\frac{15}{4}-\frac{15}{2}\right\}e\,e_{,}^2\cos(2t-x-2z)}$$

$$+\underset{[31]}{\left\{\frac{5}{4}-\frac{15}{4}+\frac{5}{2}\right\}e\,e_{,}^2\cos(2t+x+2z)}+\underset{[32]}{\left\{\frac{1}{4}+\frac{5}{2}-5\right\}e\,e_{,}^2\cos(x-2z)}$$

$$+\underset{[33]}{\left\{-\frac{15}{4}+\frac{45}{4}-\frac{15}{2}\right\}e\,e_{,}^2\cos(2t-x+2z)}+\underset{[34]}{\left\{\frac{1}{2}+\frac{5}{4}+\frac{5}{2}\right\}e\,e_{,}^2\cos(2t+x-2z)}$$

$$+\underset{[35]}{\left\{-\frac{1}{8}-\frac{5}{8}-5+\frac{145}{16}\right\}e_{,}^3\cos 3z}+\underset{[36]}{\left\{\frac{25}{48}+\frac{5}{4}+\frac{5}{2}+\frac{145}{32}\right\}e_{,}^3\cos(2t-3z)}$$

$$+\underset{[37]}{\left\{-\frac{7}{48}+\frac{25}{8}-\frac{15}{2}+\frac{145}{32}\right\}e_{,}^3\cos(2t+3z)}$$

$$-\underset{[38]}{\frac{e^4}{12}\cos 4x}-\underset{[39]}{\frac{e^4}{32}\cos(2t-4x)}+\underset{[40]}{\frac{9}{16}e^4\cos(2t+4x)}$$

$$+\underset{[41]}{\left\{\frac{1}{8}-\frac{5}{16}\right\}e^3e_{,}\cos(3x+z)}+\underset{[42]}{\left\{-\frac{7}{48}-\frac{35}{96}\right\}e^3e_{,}\cos(2t-3x-z)}$$

$$+\underset{[43]}{\left\{-\frac{25}{16}+\frac{125}{96}\right\}e^3e_{,}\cos(2t-3x-z)}+\underset{[44]}{\left\{\frac{1}{8}-\frac{5}{16}\right\}e^3e_{,}\cos(3x-z)}$$

$$+\underset{[45]}{\left\{\frac{7}{16}-\frac{35}{96}\right\} e^3 e_{,} \cos(2t-3x+z)}$$

$$+\underset{[46]}{\left\{\frac{25}{48}+\frac{125}{96}\right\} e^3 e_{,} \cos(2t+3x-z)}+\underset{[47]}{\left\{\frac{1}{16}+\frac{5}{8}-\frac{5}{4}\right\} e^2 e_{,}^2 \cos(2x+2z)}$$

$$+\underset{[48]}{\left\{\frac{5}{4}+\frac{25}{8}+\frac{25}{4}\right\} e^2 e_{,}^2 \cos(2t-2x-2z)}+\underset{[49]}{\left\{\frac{5}{4}-\frac{15}{4}+\frac{5}{2}\right\} e^2 e_{,}^2 \cos(2t+2x+2z)}$$

$$+\underset{[50]}{\left\{\frac{1}{16}+\frac{5}{8}-\frac{5}{4}\right\} e^2 e_{,}^2 \cos(2x-2z)}+\underset{[51]}{\left\{\frac{25}{8}-\frac{75}{8}+\frac{25}{4}\right\} e^2 e_{,}^2 \cos(2t-2x+2z)}$$

$$+\underset{[52]}{\left\{\frac{1}{2}+\frac{5}{4}+\frac{5}{2}\right\} e^2 e_{,}^2 \cos(2t+2x-2z)}+\underset{[53]}{\left\{\frac{1}{8}+\frac{5}{8}+5-\frac{145}{16}\right\} e\, e_{,}^3 \cos(x+3z)}$$

$$+\underset{[54]}{\left\{-\frac{25}{16}-\frac{15}{4}-\frac{15}{2}-\frac{435}{32}\right\} e\, e_{,}^3 \cos(2t-x-3z)}$$

$$+\underset{[55]}{\left\{-\frac{7}{48}+\frac{25}{8}-\frac{15}{2}+\frac{145}{32}\right\} e\, e_{,}^3 \cos(2t+x+3z)}$$

$$+\underset{[56]}{\left\{\frac{1}{8}+\frac{5}{8}+5-\frac{145}{16}\right\} e\, e_{,}^3 \cos(x-3z)}+\underset{[57]}{\left\{\frac{7}{16}-\frac{75}{8}+\frac{45}{2}-\frac{435}{32}\right\} e\, e_{,}^3 \cos(2t-x+3z)}$$

$$+\underset{[58]}{\left\{\frac{25}{48}+\frac{5}{4}+\frac{5}{2}-\frac{145}{32}\right\} e\, e_{,}^3 \cos(2t+x-3z)}-\underset{[59]}{\left\{-\frac{1}{12}-\frac{5}{16}-\frac{5}{4}-\frac{145}{16}+\frac{745}{96}\right\} e_{,}^4 \cos 4z}$$

$$+\underset{[60]}{\left\{\frac{9}{16}+\frac{125}{96}+\frac{5}{2}+\frac{145}{32}+\frac{745}{192}\right\} e_{,}^4 \cos(2t-4z)}$$

$$+\underset{[61]}{\left\{-\frac{1}{32}-\frac{35}{96}+\frac{25}{4}-\frac{435}{32}+\frac{745}{192}\right\} e_{,}^4 \cos(2t+4z)}$$

Terms in R multiplied by $-\frac{3}{2}\sin^2\frac{\iota}{2}\cos^2\frac{\iota}{2}\frac{a^2}{a_{,}^3}$

$$=\underset{[62]}{\left\{1-\frac{5}{2}e^2+\frac{3}{2}e_{,}^2+5e_{,}^2-\frac{5}{2}e_{,}^2-\frac{5}{2}e_{,}^2\right\}\cos 2y}$$

$$+\underset{[63]}{\left\{1+\frac{3}{2}e^2-\frac{5}{2}e_{,}^2+5e_{,}^2+\frac{5}{2}e_{,}^2-\frac{15}{2}e_{,}^2\right\}\cos(2t-2y)}$$

$$-\underset{[65]}{3e\cos(x-2y)}+\underset{[66]}{e\cos(x+2y)}-\underset{[67]}{e\cos(2t-x-2y)}-\underset{[69]}{e\cos(2t+x-2y)}$$

$$+\underset{[71]}{\left\{-1+\frac{5}{2}\right\}e_{,}\cos(z-2y)}+\underset{[72]}{\left\{-1+\frac{5}{2}\right\}e_{,}\cos(z+2y)}+\underset{[73]}{\left\{1+\frac{5}{2}\right\}e_{,}\cos(2t-z-2y)}$$

$$+\underset{[75]}{\left\{-3+\frac{5}{2}\right\}e_{,}\cos(2t+z-2y)}+\underset{[77]}{\frac{5}{2}e^{2}\cos(2x-2y)}+\underset{[78]}{e^{2}\cos(2x+2y)}$$

$$-\underset{[79]}{\frac{e^{2}}{4}\cos(2t-2x-2y)}-\underset{[81]}{\frac{e^{2}}{4}\cos(2t+2x-2y)}$$

$$+\underset{[83]}{\left\{3-\frac{15}{2}\right\}e\,e_{,}\cos(x+z-2y)}+\underset{[84]}{\left\{-1+\frac{5}{2}\right\}e\,e_{,}\cos(x+z+2y)}$$

$$+\underset{[85]}{\left\{-1-\frac{5}{2}\right\}e\,e_{,}\cos(2t-x-z-2y)}$$

$$+\underset{[87]}{\left\{3-\frac{5}{2}\right\}e\,e_{,}\cos(2t+x+z-2y)}+\underset{[89]}{\left\{3-\frac{15}{2}\right\}e\,e_{,}\cos(2t-z-2y)}$$

$$+\underset{[90]}{\left\{-1+\frac{5}{2}\right\}e\,e_{,}\cos(x-z+2y)}+\underset{[91]}{\left\{3-\frac{5}{2}\right\}e\,e_{,}\cos(2t-x+z-2y)}$$

$$+\underset{[93]}{\left\{-1-\frac{5}{2}\right\}e\,e_{,}\cos(2t+x-z-2y)}+\underset{[95]}{\left\{-\frac{3}{8}-\frac{5}{2}+5\right\}e_{,}^{2}\cos(2z-2y)}$$

$$+\underset{[96]}{\left\{-\frac{5}{8}-\frac{5}{2}+5\right\}e_{,}^{2}\cos(2z+2y)}+\underset{[97]}{\left\{\frac{5}{8}+\frac{5}{2}+5\right\}e_{,}^{2}\cos(2t-2z-2y)}$$

$$+\underset{[99]}{\left\{\frac{19}{8}-\frac{15}{2}+5\right\}e_{,}^{2}\cos(2t+2z-2y)}$$

Terms in R multiplied by $-\frac{3}{2}\cos^{4}\frac{\iota}{2}\,\frac{a^{2}}{a_{,}^{3}}$

$$=\frac{1}{2}+\frac{3}{4}e^{2}+\frac{3}{4}e_{,}^{2}+\frac{15}{16}e_{,}^{4}+\frac{9}{8}e^{2}e_{,}^{2}+\underset{[1]}{\left\{\frac{1}{2}-\frac{5}{4}e^{2}-\frac{5}{4}e_{,}^{2}+\frac{23}{32}e^{4}+\frac{13}{32}e_{,}^{4}+\frac{25}{8}e^{2}e_{,}^{2}\right\}\cos 2t}$$

$$+\underset{[2]}{\left\{1+\frac{e^{2}}{8}-\frac{3}{2}e_{,}^{2}\right\}e\cos x}+\underset{[3]}{\left\{-\frac{3}{2}+\frac{13}{16}e^{2}+\frac{15}{4}e_{,}^{2}\right\}e\cos(2t-x)}$$

$$+\underset{[4]}{\left\{\frac{1}{2}-\frac{19}{16}e^{2}-\frac{5}{4}e_{,}^{2}\right\}e\cos(2t+x)}+\underset{[5]}{\left\{\frac{3}{2}+\frac{9}{4}e^{2}+\frac{27}{16}e_{,}^{2}\right\}e_{,}\cos z}$$

$$+\left\{\frac{7}{4}-\frac{35}{8}e^2-\frac{123}{32}e_{,}^2\right\}\underset{[6]}{e_{,}\cos(2t-z)}$$

$$+\left\{-\frac{1}{4}+\frac{5}{8}e^2+e_{,}^2\right\}\underset{[7]}{e_{,}\cos(2t+z)}+\left\{-\frac{1}{4}+\frac{e^2}{12}-\frac{3}{8}e_{,}^2\right\}\underset{[8]}{e^2\cos 2x}$$

$$+\left\{\frac{5}{4}-\frac{25}{8}e_{,}^2\right\}\underset{[9]}{e^2\cos(2t-2x)}+\left\{\frac{1}{2}-\frac{5}{4}e^2-\frac{5}{4}e_{,}^2\right\}\underset{[10]}{e^2\cos(2t+2x)}$$

$$+\left\{-\frac{3}{2}+\frac{3}{16}e^2-\frac{27}{16}e_{,}^2\right\}\underset{[11]}{e\,e_{,}\cos(x+z)}+\left\{-\frac{21}{4}+\frac{91}{32}e^2+\frac{369}{32}e_{,}^2\right\}\underset{[12]}{e\,e_{,}\cos(2t-x-z)}$$

$$+\left\{-\frac{1}{4}+\frac{19}{32}e^2+\frac{e_{,}^2}{32}\right\}\underset{[13]}{e\,e_{,}\cos(2t+x+z)}+\left\{-\frac{3}{2}+\frac{3}{16}e^2-\frac{27}{16}e_{,}^2\right\}\underset{[14]}{e\,e_{,}\cos(x-z)}$$

$$+\left\{\frac{3}{4}-\frac{13}{32}e^2-\frac{3}{32}e_{,}^2\right\}\underset{[15]}{e\,e_{,}\cos(2t-x+z)}+\left\{\frac{7}{4}-\frac{133}{32}e^2-\frac{123}{32}e_{,}^2\right\}\underset{[16]}{e\,e_{,}\cos(2t+x-z)}$$

$$+\left\{\frac{9}{4}+\frac{27}{8}e^2+\frac{21}{12}e_{,}^2\right\}\underset{[17]}{e_{,}^2\cos 2z}+\left\{\frac{17}{4}-\frac{85}{8}e^2-\frac{115}{12}e_{,}^2\right\}\underset{[18]}{e_{,}^2\cos(2t-2z)}{}^{*}-\underset{[20]}{\frac{e^3}{8}\cos 3x}$$

$$-\underset{[21]}{\frac{7}{48}e^3\cos(2t-3x)}+\underset{[22]}{\frac{25}{48}e^3\cos(2t+3x)}-\underset{[23]}{\frac{3}{8}e^2e_{,}\cos(2x+z)}$$

$$+\underset{[24]}{\frac{35}{8}e^2e_{,}\cos(2t-2x-z)}-\underset{[25]}{\frac{e^2e_{,}}{4}\cos(2t+2x+z)}-\underset{[26]}{\frac{3}{8}e^2e_{,}\cos(2x-z)}$$

$$-\underset{[27]}{\frac{5}{8}e^2e_{,}\cos(2t-2x+z)}+\underset{[28]}{\frac{7}{4}e^2e_{,}\cos(2t+2x-z)}$$

$$-\underset{[29]}{\frac{9}{4}e\,e_{,}^2\cos(x+2z)}-\underset{[30]}{\frac{51}{4}e\,e_{,}^2\cos(2t-x-2z)}$$

$$-\underset{[32]}{\frac{9}{4}e\,e_{,}^2\cos(x-2z)}+\underset{[34]}{\frac{17}{4}e\,e_{,}^2\cos(2t+x-2z)}$$

$$+\underset{[35]}{\frac{53}{16}e_{,}^3\cos 3z}+\underset{[36]}{\frac{845}{96}e_{,}^3\cos(2t-3z)}+\underset{[37]}{\frac{e^3e_{,}}{96}\cos(2t+3z)}-\underset{[38]}{\frac{e^4}{12}\cos 4x}-\underset{[39]}{\frac{e^4}{32}\cos(2t-4x)}$$

---

* It is remarkable that the coefficient of argument 19 equals zero.

$$+\frac{9}{16}e^4\cos(2t+4x)-\frac{3}{16}e^3e_{\prime}\cos(3x+z)$$

[40] [41]

$$-\frac{49}{96}e^3e_{\prime}\cos(2t+3x-z)-\frac{25}{96}e^3e_{\prime}\cos(2t-3x-z)-\frac{3}{16}e^3e_{\prime}\cos(3x-z)$$

[42] [43] [44]

$$+\frac{7}{96}e^3e_{\prime}\cos(2t-3x+z)+\frac{175}{96}e^3e_{\prime}\cos(2t+3x-z)-\frac{9}{16}e^2e_{\prime}^2\cos(2x+2z)$$

[45] [46] [47]

$$+\frac{85}{8}e^2e_{\prime}^2\cos(2t-2x-2z)-\frac{9}{16}e^2e_{\prime}^2\cos(2x-2z)+\frac{17}{4}e^2e_{\prime}^2\cos(2t+2x-2z)$$

[48] [50] [52]

$$-\frac{53}{16}ee_{\prime}^3\cos(x+3z)-\frac{845}{32}ee_{\prime}^3\cos(2t\quad x-3z)+\frac{ee_{\prime}^3}{96}\cos(2t+x-3z)$$

[53] [54] [55]

$$-\frac{53}{16}ee_{\prime}^3\cos(x-3z)-\frac{ee_{\prime}^3}{32}\cos(2t-x+3z)-\frac{25}{96}ee_{\prime}^3\cos(2t+x-3z)$$

[56] [57] [58]

$$-\frac{283}{96}e_{\prime}^4\cos 4z+\frac{2453}{192}e_{\prime}^4\cos(2t-4z)-\frac{741}{192}e_{\prime}^4\cos(2t+4z)$$

[59] [60] [61]

Terms in R multiplied by $-\frac{3}{2}\sin^2\frac{\iota}{2}\cos^2\frac{\iota}{2}\frac{a^2}{a_{\prime}^3}$ or $-\frac{3}{8}\sin^2\iota\frac{a^2}{a_{\prime}^3}$

$$=\left\{1-\frac{5}{2}e^2+\frac{3}{2}e_{\prime}^2\right\}\cos 2y+\left\{1+\frac{3}{2}e^2-\frac{5}{2}e_{\prime}^2\right\}\cos(2t-2y)-3e\cos(x-2y)$$

[62] [63] [65]

$$+e\cos(x+2y)-e\cos(2t-x-2y)-e\cos(2t+x-2y)+\frac{3}{2}e_{\prime}\cos(z-2y)+\frac{3}{2}e_{\prime}\cos(z+2y)$$

[66] [67] [69] [71] [72]

$$+\frac{7}{2}e_{\prime}\cos(2t-z-2y)-\frac{e_{\prime}}{2}\cos(2t+z-2y)+\frac{5}{2}e^2\cos(2x-2y)+e^2\cos(2x+2y)$$

[73] [75] [77] [78]

$$-\frac{e^2}{4}\cos(2t-2x-2y)-\frac{e^2}{4}\cos(2t+2x-2y)-\frac{9}{2}ee_{\prime}\cos(x+z-2y)$$

[79] [81] [83]

$$+\frac{3}{2}ee_{\prime}\cos(x+z+2y)-\frac{7}{2}ee_{\prime}\cos(2t-x-z-2y)+\frac{ee_{\prime}}{2}\cos(2t+x+z-2y)$$

[84] [85] [87]

$$-\underset{[89]}{\frac{9}{2}ee_{,}\cos(x-z-2y)}+\underset{[90]}{\frac{3}{2}ee_{,}\cos(x-z+2y)}+\underset{[91]}{\frac{ee_{,}}{2}\cos(2t-x+z-2y)}$$

$$-\underset{[93]}{\frac{7}{2}ee_{,}\cos(2t+x-z-2y)}+\underset{[95]}{\frac{17}{8}e_{,}^{2}\cos(2z-2y)}+\underset{[96]}{\frac{15}{8}e_{,}^{2}\cos(2z+2y)}$$

$$+\underset{[97]}{\frac{65}{8}e_{,}^{2}\cos(2t-2z-2y)}-\underset{[99]}{\frac{e_{,}^{2}}{8}\cos(2t+2z-2y)}$$

$$R=m_{,}\Big\{-\frac{1}{r_{,}}-\underset{[0]}{\frac{1}{4}\Big\{1+\frac{3}{2}e^{2}+\frac{3}{2}e_{,}^{2}+\frac{9}{4}e^{2}e_{,}^{2}+\frac{15}{8}e_{,}^{4}-\frac{3}{2}\gamma^{2}-\frac{9}{4}\gamma^{2}e^{2}-\frac{9}{4}\gamma^{2}e_{,}^{2}+\frac{39}{8}\gamma^{4}\Big\}\frac{a^{2}}{a_{,}^{3}}}$$

$$-\underset{[1]}{\frac{3}{4}\Big\{1-\frac{5}{2}e^{2}-\frac{5}{2}e_{,}^{2}+\frac{23}{16}e^{4}+\frac{25}{4}e^{2}e_{,}^{2}+\frac{13}{16}e_{,}^{4}\Big\}\cos^{4}\frac{\iota}{2}\,\frac{a^{2}}{a_{,}^{3}}\cos 2t}$$

$$+\underset{[2]}{\frac{1}{2}\Big\{1-\frac{e^{2}}{8}-\frac{3}{2}e_{,}^{2}-\frac{3}{2}\gamma^{2}\Big\}\frac{a^{2}}{a_{,}^{3}}e\cos x}$$

$$+\underset{[3]}{\frac{9}{4}\Big\{1-\frac{13}{24}e^{2}-\frac{5}{2}e_{,}^{2}\Big\}\cos^{4}\frac{\iota}{2}\,\frac{a^{2}}{a_{,}^{3}}e\cos(2t-x)}$$

$$-\underset{[4]}{\frac{3}{4}\Big\{1-\frac{19}{8}e^{2}-\frac{5}{2}e_{,}^{2}\Big\}\cos^{4}\frac{\iota}{2}\,\frac{a^{2}}{a_{,}^{3}}e\cos(2t+x)}$$

$$-\underset{[5]}{\frac{3}{4}\Big\{1+\frac{3}{2}e^{2}+\frac{9}{8}e_{,}^{2}-\frac{3}{2}\gamma^{2}\Big\}\frac{a^{2}}{a_{,}^{3}}e_{,}\cos z}$$

$$-\underset{[6]}{\frac{21}{8}\Big\{1-\frac{5}{2}e^{2}-\frac{123}{56}e_{,}^{2}\Big\}\cos^{4}\frac{\iota}{2}\,\frac{a^{2}}{a_{,}^{3}}e_{,}\cos(2t-z)}$$

$$+\underset{[7]}{\frac{3}{8}\Big\{1-\frac{5}{2}e^{2}-4e_{,}^{2}\Big\}\cos^{4}\frac{\iota}{2}\,\frac{a^{2}}{a_{,}^{3}}e_{,}\cos(2t+z)}$$

$$+\underset{[8]}{\frac{1}{8}\Big\{1-\frac{e^{2}}{3}+\frac{3}{2}e_{,}^{2}-\frac{3}{2}\gamma^{2}\Big\}\frac{a^{2}}{a_{,}^{3}}e^{2}\cos 2x}$$

$$-\underset{[9]}{\frac{15}{8}\Big\{1-\frac{5}{2}e_{,}^{2}\Big\}\cos^{4}\frac{\iota}{2}\,\frac{a^{2}}{a_{,}^{3}}e^{2}\cos(2t-2x)}$$

$$-\underset{[10]}{\frac{3}{4}\Big\{1-\frac{5}{2}e^{2}-\frac{5}{2}e_{,}^{2}\Big\}\cos^{4}\frac{\iota}{2}\,\frac{a^{2}}{a_{,}^{3}}e^{2}\cos(2t+2x)}$$

Development of *R*.

$$+\frac{3}{4}\left\{1-\frac{e^2}{8}+\frac{9}{8}e_{,}^2-\frac{3}{2}\gamma^2\right\}\frac{a^2}{a_{,}^3}e\,e_{,}\cos(x+z)$$

[11]

$$+\frac{63}{8}\left\{1-\frac{91}{168}e^2-\frac{123}{56}e_{,}^2\right\}\cos^4\frac{\iota}{2}\,\frac{a^2}{a_{,}^3}e\,e_{,}\cos(2t-x-z)$$

[12]

$$+\frac{3}{8}\left\{1-\frac{19}{8}e^2-\frac{e_{,}^2}{8}\right\}\cos^4\frac{\iota}{2}\,\frac{a^2}{a_{,}^3}e\,e_{,}\cos(2t+x+z)$$

[13]

$$+\frac{3}{4}\left\{1-\frac{e^2}{8}+\frac{9}{8}e_{,}^2-\frac{3}{2}\gamma^2\right\}\frac{a^2}{a_{,}^2}e\,e_{,}\cos(x-z)$$

[14]

$$-\frac{9}{8}\left\{1-\frac{13}{24}e^2-\frac{e_{,}^2}{8}\right\}\cos^4\frac{\iota}{2}\,\frac{a^2}{a_{,}^3}e\,e_{,}\cos(2t-x+z)$$

[15]

$$-\frac{21}{8}\left\{1-\frac{19}{8}e^2-\frac{123}{56}e_{,}^2\right\}\cos^4\frac{\iota}{2}\,\frac{a^2}{a_{,}^3}e\,e_{,}\cos(2t+x-z)$$

[16]

$$-\frac{9}{8}\left\{1+\frac{3}{2}e^2+\frac{7}{9}e_{,}^2-\frac{3}{2}\gamma^2\right\}\frac{a^2}{a_{,}^3}e_{,}^2\cos 2z$$

[17]

$$-\frac{51}{8}\left\{1-\frac{5}{2}e^2-\frac{115}{51}e_{,}^2\right\}\cos^4\frac{\iota}{2}\,\frac{a^2}{a_{,}^3}e_{,}^2\cos(2t-2z)$$

[18]

$$+\frac{1}{16}\frac{a^2}{a_{,}^3}e^3\cos 3x+\frac{7a^2}{32a_{,}^3}e^3\cos(2t-3x)-\frac{25a^2}{32a_{,}^3}e^3\cos(2t+3x)$$

[20] [21] [22]

$$+\frac{3a^2}{16a_{,}^3}e^2e_{,}\cos(2x+z)-\frac{105}{16}\frac{a^2}{a_{,}^3}e^2e_{,}\cos(2t-2x-z)+\frac{3}{8}\frac{a^2}{a_{,}^3}e^2e_{,}\cos(2t+2x+z)$$

[23] [24] [25]

$$+\frac{3}{16}\frac{a^2}{a_{,}^3}e^2e_{,}\cos(2x-z)+\frac{15}{16}\frac{a^2}{a_{,}^3}e^2e_{,}\cos(2t-2x+z)-\frac{21}{8}\frac{a^2}{a_{,}^3}e^2e_{,}\cos(2t+2x-z)$$

[26] [27] [28]

$$+\frac{9}{8}\frac{a^2}{a_{,}^3}e\,e_{,}^2\cos(x+2z)+\frac{153}{8}\frac{a^2}{a_{,}^3}e\,e_{,}^2\cos(2t-x-2z)$$

[29] [30]

$$+\frac{9}{8}\frac{a^2}{a_{,}^3}e\,e_{,}^2\cos(x-2z)-\frac{51}{8}\frac{a^2}{a_{,}^3}e\,e_{,}^2\cos(2t+x-2z)-\frac{53}{32}\frac{a^2}{a_{,}^3}e_{,}^3\cos 3z$$

[32] [34] [35]

$$-\frac{845a^2}{64a_{,}^3}e_{,}^3\cos(2t-3z)-\frac{1}{64}\frac{a^2}{a_{,}^3}e_{,}^3\cos(2t+3z)+\frac{1}{24}\frac{a^2}{a_{,}^3}e^4\cos 4x$$

[36] [37] [38]

$$+\frac{3}{64}\frac{a^2}{a_{,}^3}e^4\cos(2t-4x)-\frac{27}{32}\frac{a^2}{a_{,}^3}e^4\cos(2t+4x)+\frac{3}{32}\frac{a^2}{a_{,}^3}e^3e_{,}\cos(3x+z)$$

[39] [40] [41]

Development of R.

$$+\underset{[42]}{\frac{49}{64}\frac{a^2}{a_{,}^3}e^3 e_{,}\cos(2t-3x-z)}+\underset{[44]}{\frac{25}{64}\frac{a^2}{a_{,}^3}e^3 e_{,}\cos(2t-3x-z)}+\frac{3}{32}\frac{a^2}{a_{,}^3}e^3 e_{,}\cos(3x-z)$$

$$-\underset{[45]}{\frac{7}{64}\frac{a^2}{a_{,}^3}e^3 e_{,}\cos(2t-3x+z)}-\underset{[46]}{\frac{175}{64}\frac{a^2}{a_{,}^3}e^3 e_{,}\cos(2t+3x-z)}$$

$$+\underset{[47]}{\frac{9}{32}\frac{a^2}{a_{,}^3}e^2 e_{,}^2\cos(2x+2z)}-\underset{[48]}{\frac{255}{16}e^2 e_{,}^2\cos(2t-2x-2z)}$$

$$+\underset{[50]}{\frac{9}{32}\frac{a^2}{a_{,}^3}e^2 e_{,}^2\cos(2x-2z)}-\underset{[52]}{\frac{51}{8}\frac{a^2}{a_{,}^3}e^2 e_{,}^2\cos(2t+2x-2z)}+\underset{[53]}{\frac{53}{32}\frac{a^2}{a_{,}^3}e\,e_{,}^3\cos(x+3z)}$$

$$+\underset{[54]}{\frac{2535}{64}\frac{a^2}{a_{,}^3}e\,e_{,}^3\cos(2t-x-3z)}-\underset{[55]}{\frac{1}{64}\frac{a^2}{a_{,}^3}e\,e_{,}^3\cos(2t+x+3z)}$$

$$+\underset{[56]}{\frac{53}{32}\frac{a^2}{a_{,}^3}e\,e_{,}^3\cos(x-3z)}+\underset{[57]}{\frac{3}{64}\frac{a^2}{a_{,}^3}e\,e_{,}^3\cos(2t-x+3z)}$$

$$+\underset{[58]}{\frac{45}{64}\frac{a^2}{a_{,}^3}e\,e_{,}^3\cos(2t+x-3z)}+\underset{[59]}{\frac{591}{64}\frac{a^2}{a_{,}^3}e_{,}^4\cos 4z}$$

$$-\underset{[60]}{\frac{2453}{128}\frac{a^2}{a_{,}^3}e_{,}^4\cos(2t-4z)}+\underset{[61]}{\frac{741}{128}\frac{a^2}{a_{,}^3}e_{,}^4\cos(2t+4z)}$$

$$-\underset{[62]}{\frac{3}{8}\left\{1-\frac{5}{2}e^2+\frac{3}{2}e_{,}^2\right\}\frac{a^2}{a_{,}^3}\gamma^2\cos 2y}-\underset{[63]}{\frac{3}{8}\left\{1+\frac{3}{2}e^2-\frac{5}{2}e_{,}^2+\frac{\gamma^2}{8}\right\}\frac{a^2}{a_{,}^3}\gamma^2\cos(2t-2y)}$$

$$+\underset{[65]}{\frac{9}{8}\frac{a^2}{a_{,}^3}\gamma^2 e\cos(x-2y)}$$

$$-\underset{[66]}{\frac{3}{8}\frac{a^2}{a_{,}^3}\gamma^2 e\cos(x+2y)}+\underset{[67]}{\frac{3}{8}\frac{a^2}{a_{,}^3}\gamma^2 e\cos(2t-x-2y)}$$

$$+\underset{[69]}{\frac{3}{8}\frac{a^2}{a_{,}^3}\gamma^2 e\cos(2t+x-2y)}-\underset{[71]}{\frac{9}{16}\frac{a^2}{a_{,}^3}\gamma^2 e_{,}\cos(z-2y)}$$

$$-\underset{[72]}{\frac{9}{16}\frac{a^2}{a_{,}^3}\gamma^2 e_{,}\cos(z+2y)}-\underset{[73]}{\frac{21}{16}\frac{a^2}{a_{,}^3}\gamma^2 e_{,}\cos(2t-z-2y)}$$

$$+\underset{[75]}{\frac{3}{16}\frac{a^2}{a_{,}^3}\gamma^2 e_{,}\cos(2t+z-2y)}-\underset{[77]}{\frac{15}{16}\frac{a^2}{a_{,}^3}\gamma^2 e^2\cos(2x-2y)}$$

Development of R.

$$-\underset{[78]}{\frac{3}{8}\frac{a^2}{a_{\prime}^3}\gamma^2 e^2 \cos(2x+2y)} + \underset{[79]}{\frac{3}{32}\frac{a^2}{a_{\prime}^3}\gamma^2 e^2 \cos(2t-2x-2y)}$$

$$+\underset{[81]}{\frac{3}{32}\frac{a^2}{a_{\prime}^3}\gamma^2 e^2 \cos(2t+2x-2y)} + \underset{[83]}{\frac{27}{16}\frac{a^2}{a_{\prime}^3}\gamma^2 e e_{\prime} \cos(x+z-2y)}$$

$$-\underset{[84]}{\frac{9}{16}\frac{a^2}{a_{\prime}^3}\gamma^2 e e_{\prime} \cos(x+z+2y)} + \underset{[85]}{\frac{21}{16}\frac{a^2}{a_{\prime}^3}\gamma^2 e e_{\prime} \cos(2t-x-z-2y)}$$

$$-\underset{[87]}{\frac{3}{16}\frac{a^2}{a_{\prime}^3}\gamma^2 e e_{\prime} \cos(2t+x+z-2y)} + \underset{[89]}{\frac{27}{16}\frac{a^2}{a_{\prime}^3}\gamma^2 e e_{\prime} \cos(x-z-2y)}$$

$$-\underset{[90]}{\frac{9}{16}\frac{a^2}{a_{\prime}^3}\gamma^2 e e_{\prime} \cos(x-z+2y)} - \underset{[91]}{\frac{3}{16}\frac{a^2}{a_{\prime}^2}\gamma^2 e e_{\prime} \cos(2t-x+z-2y)}$$

$$+\underset{[93]}{\frac{21}{16}\frac{a^2}{a_{\prime}^3}\gamma^2 e e_{\prime} \cos(2t+x-z-2y)}$$

$$-\underset{[95]}{\frac{51}{64}\frac{a^2}{a_{\prime}^3}\gamma^2 e_{\prime}^2 \cos(2z-2y)} - \underset{[96]}{\frac{45}{64}\frac{a^2}{a_{\prime}^3}\gamma^2 e_{\prime}^2 \cos(2z+2y)}$$

$$-\underset{[97]}{\frac{195}{64}\frac{a^2}{a_{\prime}^3}\gamma^2 e_{\prime}^2 \cos(2t-2z-2y)} + \underset{[99]}{\frac{3}{64}\frac{a^2}{a_{\prime}^3}\gamma^2 e_{\prime}^2 \cos(2t+2z-2y)}$$

$$-\underset{[101]}{\frac{3}{8}\left\{1+3e^2+3e_{\prime}^2-\frac{11}{4}\gamma^2\right\}\frac{a^3}{a_{\prime}^4}{}^{*}\cos t} + \underset{[102]}{\frac{15}{16}\frac{a^3}{a_{\prime}^4} e \cos(t-x)}$$

$$+\underset{[103]}{\frac{3}{16}\frac{a^3}{a_{\prime}^4} e \cos(t+x)} - \underset{[104]}{\frac{9}{8}\frac{a^3}{a_{\prime}^4} e_{\prime} \cos(t-z)} - \underset{[105]}{\frac{3}{8}\frac{a^3}{a_{\prime}^4} e_{\prime} \cos(t+x)}$$

$$-\underset{[106]}{\frac{33}{64}\frac{a^3}{a_{\prime}^4} e^2 \cos(t-2x)} + \underset{[107]}{\frac{9}{64}\frac{a^3}{a_{\prime}^4} e^2 \cos(t+2x)} + \underset{[108]}{\frac{45}{16}\frac{a^3}{a_{\prime}^4} e e_{\prime} \cos(t-x-z)}$$

$$+\underset{[109]}{\frac{3}{16}\frac{a^3}{a_{\prime}^4} e e_{\prime} \cos(t+x+z)} + \underset{[110]}{\frac{15}{16}\frac{a^3}{a_{\prime}^4} e e_{\prime} \cos(t-x+z)} + \underset{[111]}{\frac{9}{16}\frac{a^3}{a_{\prime}^4} e e_{\prime} \cos(t+x-z)}$$

$$-\underset{[112]}{\frac{159}{64}\frac{a^3}{a_{\prime}^4} e_{\prime}^2 \cos(t-2z)} - \underset{[113]}{\frac{33}{64}\frac{a^3}{a_{\prime}^4} e_{\prime}^2 \cos(t+2z)} - \underset{[114]}{\frac{9}{16}\frac{a^3}{a_{\prime}^4} \gamma^2 \cos(t-2y)}$$

$$-\underset{[115]}{\frac{15}{32}\frac{a^3}{a_{\prime}^4}\sin^2\frac{\iota}{2}\cos(t+2y)} - \frac{5}{8}\left\{1-6e^2-6e_{\prime}^2-\frac{3}{4}\gamma^2\right\}\frac{a^3}{a_{\prime}^4}\cos 3t \qquad [116]$$

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* For the coefficients of the terms multiplied by $\frac{a^3}{a_{\prime}^4}$ see p. 39.

$$+\underset{[117]}{\frac{45}{16}\frac{a^3}{a_{,}^4}e\cos(3t-x)}-\underset{[118]}{\frac{15}{16}\frac{a^3}{a_{,}^4}e\cos(3t+x)}-\underset{[119]}{\frac{25}{8}\frac{a^3}{a_{,}^4}e_{,}\cos(3t-z)}$$

$$+\underset{[120]}{\frac{5}{8}\frac{a^3}{a_{,}^4}e_{,}\cos(3t+z)}-\underset{[121]}{\frac{285}{64}\frac{a^3}{a_{,}^4}e^2\cos(3t-2x)}-\underset{[122]}{\frac{75}{64}\frac{a^3}{a_{,}^4}e^2\cos(3t+2x)}$$

$$-\underset{[123]}{\frac{225}{16}\frac{a^3}{a_{,}^4}ee_{,}\cos(3t-x-z)}+\underset{[124]}{\frac{15}{16}\frac{a^3}{a_{,}^4}ee_{,}\cos(3t+x+z)}$$

$$-\underset{[125]}{\frac{45}{16}\frac{a^3}{a_{,}^4}ee_{,}\cos(3t-x+z)}-\underset{[126]}{\frac{75}{16}\frac{a^3}{a_{,}^4}ee_{,}\cos(3t+x-z)}$$

$$-\underset{[127]}{\frac{635}{64}\frac{a^3}{a_{,}^4}e_{,}^2\cos(3t-2z)}-\underset{[128]}{\frac{5}{64}\frac{a^3}{a_{,}^4}e_{,}^2\cos(3t+2z)}-\underset{[129]}{\frac{15}{32}\frac{a^3}{a_{,}^4}\gamma^2\cos(3t-2y)}$$

In the elliptic movement;

$$s=\gamma\sin(g\lambda-\nu)$$

$$\lambda=nt+2e\sin x+\frac{5}{4}e^2\sin 2x$$

$$s=\underset{[146]}{\gamma(1-e^2)\sin y}+\underset{[149]}{\gamma e\sin(x-y)}+\underset{[150]}{\gamma e\sin(x+y)}+\underset{[161]}{\gamma\frac{e^2}{8}\sin(2x-y)}+\underset{[162]}{\frac{9}{8}\gamma e^2\sin(2x+y)}$$

$$s^2=\frac{\gamma^2}{2}-\underset{[62]}{\frac{\gamma^2}{2}(1-4e^2)\cos 2y}+\underset{[65]}{\gamma^2 e\cos(x-2y)}-\underset{[66]}{\gamma^2 e\cos(x+2y)}$$

$$+\underset{[77]}{\frac{5}{8}\gamma^2e^2\cos(2x-2y)}-\underset{[78]}{\frac{5}{8}\gamma^2e^2\cos(2x+2y)}$$

$$z^{*}=\underset{[146]}{a\gamma\left(1-\frac{e^2}{2}\right)\sin y}+\underset{[149]}{\frac{3a\gamma e}{2}\sin(x-y)}+\underset{[150]}{\frac{a\gamma e}{2}\sin(x+y)}$$

$$-\underset{[161]}{\frac{a\gamma e^2}{8}\sin(2x-y)}+\underset{[162]}{\frac{3a\gamma e^2}{8}\sin(2x+y)}$$

$$\frac{s}{r}=\underset{[146]}{\frac{\gamma}{a}(1-e^2)\sin y}+\underset{[149]}{\frac{\gamma e}{2a}\sin(x-y)}+\underset{[150]}{\frac{3\gamma e}{2a}\sin(x+y)}$$

---

[*] This quantity z, which is one of the rectangular coordinates of the moon, must not be confounded with $z=n_{,}t-\varpi_{,}$; this latter quantity should rather be $x_{,}$, but I think it better to conform as far as possible to the notation of M. Damoiseau.

$$+\frac{\gamma e^2}{8a}\sin(2x-y)+\frac{17}{8}\frac{\gamma e^2}{a}\sin(2x+y)$$

[161] [162]

$$\frac{s}{r}\delta\,.\,\frac{1}{r}=\left\{(1-e^2)r_0+\frac{e^2}{2}r_2\right\}\frac{\gamma}{a^2}\sin y-\left\{(1-e^2)\frac{r_1}{2}+\frac{e^2}{4}r_3-\frac{3e^2}{4}r_4\right\}\frac{\gamma}{a^2}\sin(2t-y)$$

[146] [147]

$$+\left\{(1-e^2)\frac{r_1}{2}-\frac{e^2}{4}r_4+\frac{3e^2}{4}r_3\right\}\frac{e\gamma}{a^2}\sin(2t+y)+\frac{e\gamma r_0}{2a^2}\sin(x-y)$$

[148] [149]

$$+\frac{3r_0}{2a^2}e\gamma\sin(x+y)+\left\{-\frac{r_3}{2}-\frac{3r_1}{4}\right\}\frac{e\gamma}{a^2}\sin(2t-x-y)$$

[150] [151]

$$+\left\{\frac{r_3}{2}-\frac{r_1}{4}\right\}\frac{e\gamma}{a^2}\sin(2t-x+y)+\left\{-\frac{r_4}{2}-\frac{r_1}{4}\right\}\frac{e\gamma}{a^2}\sin(2t+x-y)$$

[152] [153]

$$+\left\{\frac{r_4}{2}+\frac{3}{4}r_1\right\}\frac{e\gamma}{a^2}\sin(2t+x+y)+\frac{r_5e_,\gamma}{2a^2}\sin(z-y)+\frac{r_5e_,\gamma}{2a^2}\sin(z+y)$$

[154] [155] [156]

$$-\frac{r_6e_,\gamma}{2a^2}\sin(2t-z-y)+\frac{r_6e_,\gamma}{2a^2}\sin(2t-z+y)-\frac{r_7e_,\gamma}{2a^2}\sin(2t+z-y)$$

[157] [158] [159]

$$+\left\{-\frac{r_9}{2}-\frac{3}{4}r_3-\frac{17}{16}r_1\right\}\frac{e^2\gamma}{a^2}\sin(2t-2x-y)+\left\{\frac{r_9}{2}-\frac{r_3}{4}-\frac{r_1}{16}\right\}\frac{e^2\gamma}{a^2}\sin(2t-2x+y)$$

[163] [164)

$$+\left\{-\frac{r_{10}}{2}+\frac{r_4}{4}+\frac{r_1}{16}\right\}\frac{e^2\gamma}{a^2}\sin(2t+2x-y)+\left\{\frac{r_{10}}{2}+\frac{3r_4}{4}+\frac{17}{16}r_1\right\}\frac{e^2\gamma}{a^2}\sin(2t+2x+y)$$

[165] [166]

$$+\left\{-\frac{r_{11}}{2}+\frac{r_5}{4}\right\}\frac{ee_,\gamma}{a^2}\sin(x+z-y)+\left\{\frac{r_{11}}{2}+\frac{3r_5}{4}\right\}\frac{ee_,\gamma}{a^2}\sin(x+z+y)$$

[167] [168]

$$+\left\{-\frac{r_{12}}{2}+\frac{r_6}{4}-\frac{3}{4}r_6\right\}\frac{ee_,\gamma}{a^2}\sin(2t-x-z-y)+\left\{\frac{r_{12}}{2}+\frac{r_6}{4}\right\}\frac{ee_,\gamma}{a^2}\sin(2t-x-z+y)$$

[169] [170]

$$+\left\{-\frac{r_{13}}{2}+\frac{r_7}{4}\right\}\frac{ee_,\gamma}{a^2}\sin(2t+x+z-y)+\left\{\frac{r_{13}}{2}+\frac{3}{4}r_7\right\}\frac{ee_,\gamma}{a^2}\sin(2t+x+z+y)$$

(171) (172)

$$+\left\{-\frac{r_{14}}{2}+\frac{r_5}{2}\right\}\frac{ee_,\gamma}{a^2}\sin(x-z-y)+\left\{\frac{r_{14}}{2}+\frac{3}{4}r_5\right\}\frac{ee_,\gamma}{a^2}\sin(x-z+y)$$

(173) (174)

$$+\left\{-\frac{r_{15}}{2}-\frac{3}{4}r_7\right\}\frac{ee_,\gamma}{a^2}\sin(2t-x+z-y)+\left\{\frac{r_{15}}{2}-\frac{r_7}{2}\right\}\frac{ee_,\gamma}{a^2}\sin(2t-x+z+y)$$

[175] [176]

$$-\frac{r_{16}\,e\,e_{,}\gamma}{2\,a^2}\underset{[177]}{\sin(2\,t+x-z-y)}+\left\{\frac{r_{16}}{2}+\frac{3}{4}\,r_6\right\}\frac{e\,e_{,}\gamma}{a^2}\underset{[178]}{\sin(2\,t+x-z+y)}$$

$$-\frac{r_{17}}{2}\,\frac{e_{,}^{2}\gamma}{a^2}\underset{[179]}{\sin(2\,z-y)}+\frac{r_{17}}{2}\,\frac{e_{,}^{2}\gamma}{a^2}\underset{[180]}{\sin(2\,z+y)}-\frac{r_{18}}{2}\,\frac{e_{,}^{2}\gamma}{a^2}\underset{[181]}{\sin(2\,t-2\,z-y)}$$

$$+\frac{r_{18}}{2}\,\frac{e_{,}^{2}\gamma}{a^2}\underset{[182]}{\sin(2\,t-2\,z+y)}-\frac{r_{19}}{2}\,\frac{e_{,}^{2}\gamma}{a^2}\underset{[183]}{\sin(2\,t+2\,z-y)}+\frac{r_{19}}{2}\,\frac{e_{,}^{2}\gamma}{a^2}\underset{[184]}{\sin(2\,t+2\,z+y)}$$

$$\frac{m_{,}\,z}{r^3}=\frac{m_{,}\,a\,\gamma}{a_{,}^{3}}\left(1+\frac{3}{2}\,e_{,}^{2}-\frac{e^2}{2}\right)\underset{[146]}{\sin y}+\frac{3\,m_{,}\,a\,\gamma\,e}{2\,a_{,}^{3}}\underset{[149]}{\sin(x-y)}+\frac{m_{,}\,a\,\gamma}{2\,a_{,}^{3}}\underset{[150]}{\sin(x+y)}$$

$$-\frac{3\,m_{,}\,a\,\gamma\,e_{,}}{2\,a_{,}^{3}}\underset{[155]}{\sin(z-y)}+\frac{3\,m_{,}\,a\,\gamma\,e_{,}}{2\,a_{,}^{3}}\underset{[156]}{\sin(z+y)}-\frac{m_{,}\,a\,\gamma\,e^2}{8\,a_{,}^{3}}\underset{[161]}{\sin(2\,x-y)}$$

$$+\frac{3\,m_{,}\,a\,\gamma\,e^2}{8\,a_{,}^{3}}\underset{[162]}{\sin(2\,x+y)}+\frac{9\,m_{,}\,a\,\gamma\,e\,e_{,}}{4\,a_{,}^{3}}\underset{[167]}{\sin(x+z-y)}+\frac{3\,m_{,}\,a\,\gamma\,e\,e_{,}}{4\,a_{,}^{3}}\underset{[168]}{\sin(x+z+y)}$$

$$+\frac{9\,m_{,}\,a\,\gamma\,e\,e_{,}}{4\,a_{,}^{3}}\underset{[173]}{\sin(x-z-y)}+\frac{3\,m_{,}\,a\,\gamma\,e\,e_{,}}{4\,a_{,}^{3}}\underset{[174]}{\sin(x-z+y)}-\frac{9\,m_{,}\,a\,\gamma\,e_{,}^{2}}{4\,a_{,}^{3}}\underset{[179]}{\sin(2\,z-y)}$$

$$+\frac{9\,m_{,}\,a\,\gamma\,e_{,}^{2}}{4\,a_{,}^{3}}\underset{[180]}{\sin(2\,z+y)}$$

$$\frac{a^3}{\mathrm{r}^3}=1+\frac{3}{2}\,e^2+3\,e\cos x+\frac{9}{2}\,e^2\cos 2\,x$$

r being the elliptic value of r.

If $z=a\,\gamma\,z_{146}\sin y+a\,\gamma\,z_{147}\sin(2\,t-y)+a\,\gamma\,z_{148}\sin(2\,t+y)$ &c.

$$\frac{z}{r^3}{}^{*}=\left\{\left(1+\frac{3\,e^2}{2}\right)z_{146}+\frac{3}{2}\,e^2 z_{150}-\frac{3\,e^2}{2}\,z_{149}\right\}\frac{\gamma}{a^2}\underset{[146]}{\sin y}$$

$$+\left\{\left(1+\frac{3\,e^2}{2}\right)z_{147}+\frac{3\,e^2}{2}\,z_{151}+\frac{3\,e^2}{2}\,z_{153}\right\}\frac{\gamma}{a^2}\underset{[147]}{\sin(2\,t-y)}$$

$$+\left\{\left(1+\frac{3\,e^2}{2}\right)z_{148}+\frac{3\,e^2}{2}\,z_{152}+\frac{3\,e^2}{2}\,z_{154}\right\}\frac{\gamma}{a^2}\underset{[148]}{\sin(2\,t+y)}$$

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* This multiplication of z by r^{-3} may be effected at once by means of Table II.

$$+ \left\{ z_{149} - \frac{3}{2} z_{146} \right\} \frac{e\gamma}{a^2} \underset{[149]}{\sin (x - y)} + \left\{ z_{150} + \frac{3}{2} z_{146} \right\} \frac{e\gamma}{a^2} \underset{[150]}{\sin (x + y)}$$

$$+ \left\{ z_{151} + \frac{3}{2} z_{147} \right\} \frac{e\gamma}{a^2} \underset{[151]}{\sin (2t - x - y)} + \left\{ z_{152} + \frac{3}{2} z_{148} \right\} \frac{e\gamma}{a^2} \underset{[152]}{\sin (2t - x + y)}$$

$$+ \left\{ z_{153} + \frac{3}{2} z_{147} \right\} \frac{e\gamma}{a^2} \underset{[153]}{\sin (2t + x - y)}$$

$$+ \left\{ z_{154} + \frac{3}{2} z_{148} \right\} \frac{e\gamma}{a^2} \underset{[154]}{\sin (2t + x + y)} + z_{155} \frac{e_{\prime}\gamma}{a^2} \underset{[155]}{\sin (z - y)} + z_{156} \frac{e_{\prime}\gamma}{a^2} \underset{[156]}{\sin (z + y)}$$

$$+ \left\{ z_{161} + \frac{3}{2} z_{149} - \frac{9}{4} z_{146} \right\} \frac{e^2\gamma}{a^2} \underset{[161]}{\sin (2x - y)} + \left\{ z_{162} + \frac{3}{2} z_{150} + \frac{9}{4} z_{146} \right\} \frac{e^2\gamma}{a^2} \underset{[162]}{\sin (2x + y)}$$

$$+ \left\{ z_{163} + \frac{3}{2} z_{151} + \frac{9}{4} z_{147} \right\} \frac{e^2\gamma}{a^2} \underset{[163]}{\sin (2t - 2x - y)}$$

$$+ \left\{ z_{164} + \frac{3}{2} z_{152} + \frac{9}{4} z_{148} \right\} \frac{e^2\gamma}{a^2} \underset{[164]}{\sin (2t - 2x + y)}$$

$$+ \left\{ z_{165} + \frac{3}{2} z_{153} + \frac{9}{4} z_{147} \right\} \frac{e^2\gamma}{a^2} \underset{[165]}{\sin (2t + 2x - y)}$$

$$+ \left\{ z_{166} + \frac{3}{2} z_{154} + \frac{9}{4} z_{148} \right\} \frac{e^2\gamma}{a^2} \underset{[166]}{\sin (2t + 2x + y)} + \&c.$$

$$+ \left\{ z_{167} + \frac{3}{2} z_{155} \right\} \frac{e e_{\prime}\gamma}{a^2} \underset{[167]}{\sin (x + z - y)} + \left\{ z_{168} + \frac{3}{2} z_{156} \right\} \frac{e e_{\prime}\gamma}{a^2} \underset{[168]}{\sin (x + z + y)}$$

$$+ \left\{ z_{169} + \frac{3}{2} z_{157} \right\} \frac{e e_{\prime}\gamma}{a^2} \underset{[169]}{\sin (2t - x - z - y)} + \left\{ z_{170} + \frac{3}{2} z_{158} \right\} \frac{e e_{\prime}\gamma}{a^2} \underset{[170]}{\sin (2t - x - z + y)}$$

$$+ \left\{ z_{171} + \frac{3}{2} z_{159} \right\} \frac{e e_{\prime}\gamma}{a^2} \underset{[171]}{\sin (2t + x + z - y)} + \left\{ z_{172} + \frac{3}{2} z_{160} \right\} \frac{e e_{\prime}\gamma}{a^2} \underset{[172]}{\sin (2t + x + z + y)}$$

$$+ \left\{ z_{173} - \frac{3}{2} z_{156} \right\} \frac{e e_{\prime}\gamma}{a^2} \underset{[173]}{\sin (x - z - y)} + \left\{ z_{174} - \frac{3}{2} z_{155} \right\} \frac{e e_{\prime}\gamma}{a^2} \underset{[174]}{\sin (x - z + y)}$$

$$+ \left\{ z_{175} + \frac{3}{2} z_{159} \right\} \frac{e e_{\prime}\gamma}{a^2} \underset{[175]}{\sin (2t - x + z - y)} + \left\{ z_{176} + \frac{3}{2} z_{160} \right\} \frac{e e_{\prime}\gamma}{a^2} \underset{[176]}{\sin (2t - x + z + y)}$$

$$+ \left\{ z_{177} + \frac{3}{2} z_{157} \right\} \frac{e e_{\prime}\gamma}{a^2} \underset{[177]}{\sin (2t + x - z - y)} + \left\{ z_{178} + \frac{3}{2} z_{158} \right\} \frac{e e_{\prime}\gamma}{a^2} \underset{[178]}{\sin (2t + x - z + y)}$$

$s = \frac{z}{r}$ nearly,

$$= \left\{z_{146} + \frac{e^2}{2} z_{150} + \frac{e^2}{2} z_{149}\right\} \gamma \sin y$$

$$+ \left\{z_{147} + \frac{e^2}{2} z_{151} + \frac{e^2}{2} z_{153}\right\} \gamma \sin (2t - y)$$

$$+ \left\{z_{148} + \frac{e^2}{2} z_{152} + \frac{e^2}{2} z_{154}\right\} \gamma \sin (2t + y) + \&c.$$

$$\frac{\mathrm{d}^2 . r^2}{2 . \mathrm{d} t^2} - \frac{\mu}{r} + \frac{\mu}{a} + 2 \int \mathrm{d} R + r \left(\frac{\mathrm{d} R}{\mathrm{d} r}\right) = 0$$

$$\frac{\mathrm{d}^2 z}{\mathrm{d} t^2} + \frac{\mu z}{r^3} + \frac{m_{,} z}{\{r^2 - 2 r r' \cos (\lambda - \lambda) + r_{,}^2\}^{\frac{3}{2}}}$$

$$r'^4 . \frac{\mathrm{d} \lambda'^2}{\mathrm{d} t^2} = h^2 - 2 \int r'^2 \left(\frac{\mathrm{d} R}{\mathrm{d} \lambda'}\right) \mathrm{d} \lambda'$$

Neglecting the square of the disturbing force

$$\frac{- \mathrm{d}^2 . \mathrm{r}^3 \delta . \frac{1}{r}}{\mathrm{d} t^2} - \mu \delta . \frac{1}{r} + 2 \int \mathrm{d} R + r \left(\frac{\mathrm{d} R}{\mathrm{d} r}\right) = 0$$

$$\frac{\mathrm{d}^2 z}{\mathrm{d} t^2} + \frac{\mu z}{r^3} + \frac{m_{,} z}{r_{,}^3} + \frac{3 m_{,} z r' r \cos (\lambda' - \lambda)}{r_{,}^5} = 0$$

$$\frac{\mathrm{d}^2 . \delta z}{\mathrm{d} t^2} + \frac{3 \mu s \delta . \frac{1}{r}}{\mathrm{r}} + \frac{\mu \delta . z}{\mathrm{r}^3} + \frac{m_{,} z}{r_{,}^3} + \frac{3 \mu_{,} z r' r \cos (\lambda' - \lambda)}{r_{,}^5} = 0$$

$$\frac{\mathrm{d} \lambda'}{\mathrm{d} t} = \frac{h (1 + s^2)}{r^2} - \frac{(1 + \mathrm{s}^2)}{\mathrm{r}^2} \int \left(\frac{\mathrm{d} R}{\mathrm{d} \lambda'}\right) \mathrm{d} t$$

$r \left(\frac{\mathrm{d} R}{\mathrm{d} r}\right) = a \left(\frac{\mathrm{d} R}{\mathrm{d} a}\right)$, $\frac{\mathrm{d} R}{\mathrm{d} \lambda'} = \frac{\mathrm{d} R}{\mathrm{d} t}$, ($t$ being used for $n t - n_{,} t$).

Integrating the equation of p. 270, line 9, by the method of indeterminate coefficients, neglecting the cubes and higher powers of e in order to obtain a first approximation, m being equal to $\frac{n_{,}}{n}$ as in the notation of M. DAMOISEAU;

$$- r_0 - \frac{m_{,} a^3}{2 \mu a_{,}^3} \left\{1 + \frac{3}{2} e^2 + \frac{3}{2} e_{,}^2 - \frac{3}{2} \gamma^2\right\} = 0$$

$$4 (1 - m)^2 \left\{(1 + 3 e^2) r_1 - \frac{3 e^2}{2} \{r_3 + r_4\}\right\} - r_1$$

$$-\frac{3\,m_{\prime}\,a^3}{2\,\mu\,a_{\prime}^{\,3}}\left\{1-\frac{5}{2}\,e^2-\frac{5}{2}\,e_{\prime}^{\,2}-\frac{\gamma^2}{2}\right\}\left\{\frac{1}{1-m}+1\right\}=0$$

$$c^{2}{}^{*}\{1-3\,r_0\}-1+\frac{2\,m_{\prime}\,a^3}{\mu\,a^3}=0$$

$$(2-2\,m-c)^2\left\{r_3-\frac{3}{2}\,r_1\right\}-r_3+\frac{9}{2}\,\frac{m_{\prime}}{\mu}\,\frac{a^3}{a_{\prime}^{\,3}}\left\{\frac{2-c}{2-2\,m-c}+1\right\}=0$$

$$(2-2\,m+c)^2\left\{r_4-\frac{3}{2}\,r_1\right\}-r_4-\frac{3}{2}\,\frac{m_{\prime}}{\mu}\,\frac{a^3}{a_{\prime}^{\,3}}\left\{\frac{2+c}{2-2\,m+c}+1\right\}=0$$

$$m^2\,r_5-r_5-\frac{3}{2}\,\frac{m_{\prime}}{\mu}\,\frac{a^3}{a_{\prime}^{\,3}}=0$$

$$(2-3\,m)^2\,r_6-r_6-\frac{21}{4}\,\frac{m_{\prime}}{\mu}\,\frac{a^3}{a_{\prime}^{\,3}}\left\{\frac{2}{2-3\,m}+1\right\}=0$$

$$(2-m)^2\,r_7-r_7+\frac{3}{4}\,\frac{m_{\prime}}{\mu}\,\frac{a^3}{a_{\prime}^{\,3}}\left\{\frac{2}{2-m}+1\right\}=0$$

$$4\,c^2\left\{(1-3\,r_0)\,r_8-\frac{3}{4}+3\,r_0\right\}-r_8+\frac{m_{\prime}}{2\,\mu}\,\frac{a^3}{a_{\prime}^{\,3}}=0$$

$$(2-2\,m-2\,c)^2\left\{r_9-\frac{3}{2}\,r_3\right\}-r_9-\frac{15}{4}\,\frac{m_{\prime}}{\mu}\,\frac{a^3}{a_{\prime}^{\,3}}\left\{\frac{2-2\,c}{2-2\,m-2\,c}+1\right\}=0$$

$$(2-2\,m+2\,c)^2\left\{r_{10}-\frac{3}{2}\,r_4\right\}-r_{10}-\frac{3}{2}\,\frac{m_{\prime}}{\mu}\,\frac{a^3}{a_{\prime}^{\,3}}\left\{\frac{2+2\,c}{2-2\,m+2\,c}+1\right\}=0$$

$$(c+m)^2\left\{r_{11}-\frac{3}{2}\,r_5\right\}-r_{11}+\frac{3}{2}\,\frac{m_{\prime}}{\mu}\,\frac{a^3}{a_{\prime}^{\,3}}\left\{\frac{c}{c+m}+1\right\}=0$$

$$(2-3\,m-c)^2\left\{r_{12}-\frac{3}{2}\,r_6\right\}-r_{12}+\frac{63}{4}\,\frac{m_{\prime}}{\mu}\,\frac{a^3}{a_{\prime}^{\,3}}\left\{\frac{c}{2-3\,m-c}+1\right\}=0$$

$$(2-m+c)^2\left\{r_{13}-\frac{3}{2}\,r_7\right\}-r_{13}+\frac{3}{4}\,\frac{m_{\prime}}{\mu}\,\frac{a^2}{a_{\prime}^{\,3}}\left\{\frac{2+c}{2-m+c}+1\right\}=0$$

$$(c-m)^2\left\{r_{14}-\frac{3}{2}\,r_5\right\}-r_{14}+\frac{3}{2}\,\frac{m_{\prime}}{\mu}\,\frac{a^3}{a_{\prime}^{\,3}}\left\{\frac{c}{c-m}+1\right\}=0$$

$$(2-m-c)^2\left\{r_{15}-\frac{3}{2}\,r_7\right\}-r_{15}-\frac{9}{4}\,\frac{m_{\prime}}{\mu}\,\frac{a^3}{a_{\prime}^{\,3}}\left\{\frac{2-c}{2-m-c}+1\right\}=0$$

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[*] The letter *c* does not strictly denote the same quantity as in the notation of M. DAMOISEAU, or in that of the Mathematical Tracts, p. 33.

$$(2-3m+c)^2\left\{r_{16}-\frac{3}{2}r_6\right\}-r_{16}-\frac{21}{4}\frac{m_,}{\mu}\frac{a^3}{a_,^3}\left\{\frac{2+c}{2-3m+c}+1\right\}=0$$

$$4m^2r_{17}-r_{17}-\frac{9}{4}\frac{m_,}{\mu}\frac{a^3}{a_,^3}=0$$

$$(2-4m)^2r_{18}-r_{18}-\frac{51}{4}\frac{m_,}{\mu}\frac{a^3}{a_,^3}\left\{\frac{2}{2-4m}+1\right\}$$

$$4r_{19}-r_{19}=0$$

The equation for determining z may be integrated in the same way.

$$-g^2z_{146}+3r_0+z_{146}+\frac{m_,}{\mu}\frac{a^3}{a_,^3}=0$$

$$-\left\{2(1-m)-g\right\}^2z_{147}-\frac{3r_1}{2}+z_{147}=0$$

$$-\left\{2(1-m)+g\right\}^2z_{148}+\frac{3r_1}{2}+z_{148}=0$$

$$-\left\{c-g\right\}^2z_{149}+\frac{3}{2}r_6+z_{149}-\frac{3}{2}z_{146}+\frac{3m_,}{2\mu}\frac{a^3}{a_,^3}=0$$

$$-\left\{c+g\right\}^2z_{150}+\frac{9}{2}r_6+z_{150}+\frac{3}{2}z_{146}+\frac{m_,}{\mu}\frac{a^3}{a_,^3}=0$$

$$-\left\{2(1-m)-c-g\right\}^2z_{147}+3\left\{-\frac{3r_1}{4}-\frac{r_3}{2}\right\}+z_{151}+\frac{3}{2}z_{147}=0$$

$$-\left\{2(1-m)-c+g\right\}^2z_{148}+3\left\{-\frac{r_1}{4}+\frac{r_3}{2}\right\}+z_{152}+\frac{3}{2}z_{148}=0$$

$$-\left\{2(1-m)+c-g\right\}^2z_{149}+3\left\{\frac{r_1}{4}-\frac{r_4}{2}\right\}+z_{153}+\frac{3}{2}z_{147}=0$$

$$-\left\{2(1-m)+c+g\right\}^2z_{150}+3\left\{\frac{3}{4}r_1+\frac{r_4}{2}\right\}+z_{154}+\frac{3}{2}z_{148}=0$$

$$-\left\{m-g\right\}^2z_{151}+\frac{3}{2}r_5+z_{155}-\frac{3m_,}{2\mu}\frac{a^3}{a_,^3}=0$$

$$-\left\{m+g\right\}^2z_{152}+\frac{3}{2}r_5+z_{156}+\frac{3m_,}{2\mu}\frac{a^3}{a_,^3}=0$$

$$-\left\{2(1-m)-m-g\right\}^2z_{153}-\frac{3}{2}r_6+z_{157}=0$$

$$-\left\{2(1-m)-m+g\right\}^2z_{154}+\frac{3}{2}r_6+z_{158}=0$$

$$-\left\{2(1-m)+m-g\right\}^2z_{155}-\frac{3}{2}r_7+z_{159}=0$$

$$-\left\{2(1-m)+m+g\right\}^2 z_{156}+\frac{3}{2}r_7+z_{160}=0$$

$$\frac{d\lambda'}{dt}=\frac{h}{\mathrm{r}^2}+\frac{2h}{\mathrm{r}}\delta.\frac{1}{r}+\frac{hz^2}{r^4}-\frac{(1+s^2)}{\mathrm{r}^2}\int\left(\frac{dR}{d\lambda'}\right)dt$$

$$\lambda'=\frac{h}{a^2}\left\{1+\frac{e^2}{2}+\frac{\gamma^2}{2}+2r_0\right\}t+\frac{2e(1+r_0)}{c}\sin x+\frac{5e^2(1+r_0)}{4c}\sin 2x$$

$$+\left\{2r_1+e^2(r_3+r_4)-\left\{-\left(1-\frac{5}{2}e^2-\frac{5}{2}e_{\prime}^2-\frac{\gamma^2}{2}\right)\frac{3}{4(1-m)}+\frac{9e^2}{2(2-2m-c)}\right.\right.$$

$$\left.\left.-\frac{3e^2}{2(2-2m+c)}\right\}\frac{m_{\prime}}{\mu}\frac{a^3}{a_{\prime}^3}\right\}\frac{1}{2(1-m)}\sin 2t$$

$$+\left\{2r_3+e^2r_1-\left\{\frac{9}{2(2-m-c)}-\frac{3}{2(2-m)}\right\}\frac{m_{\prime}a^3}{\mu a_{\prime}^3}\right\}\frac{e}{(2-2m-c)}\sin(2t-x)$$

$$+\left\{2r_4+e^2r_1-\left\{-\frac{3}{2(2-m+c)}-\frac{3}{2(2-m)}\right\}\frac{m_{\prime}}{\mu}\frac{a^3}{a_{\prime}^3}\right\}\frac{e}{(2-m+c)}\sin(2t+x)$$

$$+\frac{2r_5}{m}\sin z$$

$$+\left\{2r_6+\frac{21}{4(2-3m)}\frac{m_{\prime}}{\mu}\frac{a^3}{a_{\prime}^3}\right\}\frac{e_{\prime}}{(2-3m)}\sin(2t-z)$$

$$+\left\{2r_7-\frac{3}{4(2-m)}\frac{m_{\prime}}{\mu}\frac{a^3}{a_{\prime}^3}\right\}\frac{e_{\prime}}{(2-m)}\sin(2t+z)$$

$$+\left\{2r_9+r_3-\left\{-\frac{15}{4(2-2m-2c}+\frac{9}{2(2-2m-c}\right\}\frac{m_{\prime}}{\mu}\frac{a^3}{a_{\prime}^3}\right\}\frac{e^2}{2(1-m-c)}\sin(2t-2x)$$

$$+\left\{2r_{10}+r_4-\left\{-\frac{3}{2(2-2m+2c)}-\frac{3}{2(2-m+c)}\right\}\frac{m_{\prime}}{\mu}\frac{a^3}{a_{\prime}^3}\right\}\frac{e^2}{2(1-m+c)}\sin(2t+2x)$$

$$+\left\{2r_{11}+r_5\right\}\frac{ee_{\prime}}{(c+m)}\sin(x+z)$$

$$+\left\{2r_{12}+r_6-\left\{\frac{63}{4(2-3m-c)}-\frac{21}{4(2-3m)}\right\}\frac{m_{\prime}}{\mu}\frac{a^3}{a_{\prime}^3}\right\}\frac{ee_{\prime}}{(2-3m-c)}\sin(2t-x-z)$$

$$+\left\{2r_{13}+r_7-\left\{\frac{3}{4(2-m+c)}+\frac{3}{4(2-m)}\right\}\frac{m_{\prime}}{\mu}\frac{a^3}{a_{\prime}^3}\right\}\frac{ee_{\prime}}{(2-m+c)}\sin(2t+x+z)$$

Considering the terms which depend on the square of the disturbing force

$$\frac{d^2.r^2}{2\,dt^2}-\frac{\mu}{r}+\frac{\mu}{a}+2\int dR+r\left(\frac{dR}{dr}\right)=0$$

$$\frac{\mathrm{d}^2 . \mathrm{r}^2}{2\,\mathrm{d}\,t^2} - \frac{\mathrm{d}^2 . \mathrm{r}^3\,\delta . \frac{1}{r}}{\mathrm{d}\,t^2} + \frac{3\,\mathrm{d}^2 . \mathrm{r}^4 \left(\delta . \frac{1}{r}\right)^2}{2\,\mathrm{d}\,t^2} - \frac{\mu}{r} + \frac{\mu}{a} + 2\int \mathrm{d}\,R + r\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,r}\right) = 0$$

$$\frac{\mathrm{d}^2 z}{\mathrm{d}\,t^2} + \frac{\mu\,z}{r^3} + \frac{m_{,}\,z}{r_{,}^{3}} - \frac{3\,m_{,}\,z\,r\;r\cos\,(\lambda' - \lambda)}{r_{,}^{5}} = 0.$$

$$\frac{\mathrm{d}\,\lambda^{\backprime}}{\mathrm{d}\,t} = \frac{h}{r^{\backprime 2}}\left\{1 - \frac{1}{h}\int\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,\lambda^{\backprime}}\right)\mathrm{d}\,t\left\{1 - \frac{1}{h^2}\int\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,\lambda^{\backprime}}\right)\mathrm{d}\,t\right\} - \frac{1}{2\,h^2}\left\{\int\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,\lambda^{\backprime}}\right)\mathrm{d}\,t\right\}^2\right.$$

$$= \frac{h\,(1 + s^2)}{r^2} - \frac{(1 + s^2)}{r^2}\int\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,\lambda^{\backprime}}\right)\mathrm{d}\,t + \frac{(1 + s^2)}{2\,\mathrm{r}^2\,h}\left\{\int\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,\lambda^{\backprime}}\right)\mathrm{d}\,t\right\}^2$$

d R = the differential of R, supposing $n\,t$ only variable + the differential of R, with regard to $n_{,}t$ only in as much as it is contained in the terms in r, λ and s due to the perturbations; hence

d R = the differential of R, supposing only $n\,t$ variable $+ \frac{\mathrm{d}R}{\mathrm{d}\,r^{\backprime}} . \mathrm{d} . \delta\,r^{\backprime} + \frac{\mathrm{d}\,R}{\mathrm{d}\,r}\,\mathrm{d} . \delta\,\lambda^{\backprime} + \frac{\mathrm{d}\,R}{\mathrm{d}\,r}\,\mathrm{d} . \delta\,s$; $\mathrm{d} . \delta\,r^{\backprime}$, $\mathrm{d} . \delta\,\lambda$, and $\mathrm{d} . \delta\,s$, being restrained to mean the differentials of those quantities with regard to $n_{,}t$ only.

$$\delta\,R = \left(\frac{\mathrm{d}\,R}{\mathrm{d}\,r^{\backprime}}\right)\delta\,r^{\backprime} + \left(\frac{\mathrm{d}\,R}{\mathrm{d}\,\lambda^{\backprime}}\right)\delta\,\lambda^{\backprime} + \left(\frac{\mathrm{d}\,R}{\mathrm{d}\,s}\right)\delta\,s = -\,a\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,a}\right)r^{\backprime}\,\delta . \frac{1}{r^{\backprime}} + \left(\frac{\mathrm{d}\,R}{\mathrm{d}\,t}\right)\delta\,\lambda^{\backprime} + \left(\frac{\mathrm{d}\,R}{\mathrm{d}\,s}\right)\delta\,s,$$

(t being used in the sense $n\,t - n_{,}t$.) $\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,s}\right)\delta\,s = \frac{r^2}{2\,r_{,}^{3}}\,s\,\delta\,s$ nearly.

$$\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,r}\right)\mathrm{d} . \delta\,r + \left(\frac{\mathrm{d}\,R}{\mathrm{d}\,\lambda^{\backprime}}\right)\mathrm{d} . \delta\,\lambda^{\backprime} + \left(\frac{\mathrm{d}\,R}{\mathrm{d}\,s}\right)\mathrm{d} . \delta\,s = -\,a\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,a}\right)\mathrm{d} . \mathrm{r}\,\delta . \frac{1}{r} + \left(\frac{\mathrm{d}\,R}{\mathrm{d}\,t}\right)\mathrm{d} . \delta\,\lambda^{\backprime} + \left(\frac{\mathrm{d}\,R}{\mathrm{d}\,s}\right)\mathrm{d} . \delta\,s$$

$\mathrm{d} . r^{\backprime}\,\delta\frac{1}{r^{\backprime}}$, $\mathrm{d} . \delta\,\lambda^{\backprime}$ and $\mathrm{d} . \delta\,s$ being restrained to mean the differentials of those quantities with regard to $n_{,}t$ only.

$$\delta . r\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,r}\right) = \mathrm{d} . \frac{r\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,r}\right)}{\mathrm{d}\,r^{\backprime}} . \delta\,r^{\backprime} + \mathrm{d} . \frac{r\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,r}\right)}{\mathrm{d}\,\lambda^{\backprime}}\,\delta\,\lambda^{\backprime} + \mathrm{d} . \frac{r\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,r}\right)}{\mathrm{d}\,s}\,\delta\,s$$

$$= -\,a\,\mathrm{d} . \frac{r\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,r}\right)}{\mathrm{d}\,a}\,\mathrm{r}^{\backprime} . \delta . \frac{1}{r^{\backprime}} + \mathrm{d} . \frac{r\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,r}\right)}{\mathrm{d}\,t} . \delta\,\lambda^{\backprime} + \mathrm{d} . \frac{r\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,r}\right)}{\mathrm{d}\,s}\,\delta\,s$$

$$\delta . \left(\frac{\mathrm{d}\,R}{\mathrm{d}\,\lambda^{\backprime}}\right) = -\,a . \mathrm{d} . \frac{\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,\lambda^{\backprime}}\right)}{\mathrm{d}\,a}\,\mathrm{r}^{\backprime}\,\delta . \frac{1}{r^{\backprime}} + \mathrm{d} . \frac{\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,\lambda}\right)}{\mathrm{d}\,t}\,\delta\,\lambda^{\backprime} + \frac{\left(\frac{\mathrm{d}\,R}{\mathrm{d}\,\lambda^{\backprime}}\right)}{\mathrm{d}\,s}\,\delta\,s$$

A similar theorem exists with the quantity $\delta . \frac{\mathrm{d} R}{\mathrm{d} z}$, and it will readily be seen that all the developments δR, $\delta . r' \left(\frac{\mathrm{d} R}{\mathrm{d} r'}\right)$, $\delta . \left(\frac{\mathrm{d} R}{\mathrm{d} \lambda'}\right)$ and $\delta . \left(\frac{\mathrm{d} R}{\mathrm{d} z}\right)$ may be effected very easily by means of Table II.

Similarly, if δ' denote the variation due to the disturbance of the earth by the moon,

$$\delta' R = - a_{,} \mathrm{d} . \left(\frac{\mathrm{d} R}{\mathrm{d} a_{,}}\right) r_{,} \delta . \frac{1}{r_{,}} - \mathrm{d} . \left(\frac{\mathrm{d} R}{\mathrm{d} t}\right) \delta \lambda_{,}$$

In $\mathrm{d} R$ the terms which arise from

$$- a \left(\frac{\mathrm{d} R}{\mathrm{d} a}\right) \mathrm{d} . r' \delta . \frac{1}{r'} + \left(\frac{\mathrm{d} R}{\mathrm{d} t}\right) \mathrm{d} . \delta \lambda' + \left(\frac{\mathrm{d} R}{\mathrm{d} s}\right) \mathrm{d} . \delta s$$

are multiplied by the small quantity m.

Considering in $r' \left(\frac{\mathrm{d} R}{\mathrm{d} r'}\right)$ and R the terms multiplied by $\frac{a^2}{a_{,}^3}$,

$$r' \left(\frac{\mathrm{d} R}{\mathrm{d} r'}\right) = 2 R, \qquad \delta . r' \left(\frac{\mathrm{d} R}{\mathrm{d} r'}\right) = 2 \delta R ;$$

considering the terms multiplied by $\frac{a^3}{a_{,}^4}$,

$$r' \left(\frac{\mathrm{d} R}{\mathrm{d} r'}\right) = 3 R, \qquad \delta . r' \left(\frac{\mathrm{d} R}{\mathrm{d} r'}\right) = 3 \delta R$$

Hence the value of $r' \left(\frac{\mathrm{d} R}{\mathrm{d} r'}\right)$ and $\delta . r' \left(\frac{\mathrm{d} R}{\mathrm{d} r'}\right)$ may at once be inferred from R and δR.

I reserve the formation of these developments and of the final equations for determining the coefficients of the different inequalities to another opportunity. These equations are voluminous when all sensible quantities are taken into account; but they are formed with so much facility by means of Table II., that error is not likely to arise in this part of the process. Error is more, I think, to be apprehended in the terms of R multiplied by the cubes and fourth powers of the eccentricities; the rest have been verified by an independent method. See p. 39.

Addition to Table I.

	146	149	150	
1	148 147	153 152	154 151	1
2	150 149	161 162	162 –146	2
3	152 151	147 164	148 163	3
4	154 153	165 148	166 147	4
5	156 155	167 –173	168 –174	5
6	158 157	169 170	178 169	6
7	160 159	171 176	172 175	7
146	62 0	2 – 65	66 – 2	146
147	1 63	69 3	4 67	147
148	64 1	4 68	70 3	148
149	2 65	77 0	8 – 62	149
150	66 2	8 62	78 0	150
151	3 67	63 9	1 79	151
152	68 3	1 80	64 9	152
153	4 69	81 1	10 63	153
154	70 4	10 64	82 1	154
155	5 71	83 – 14	11 – 90	155
156	72 5	11 – 89	84 – 14	156
157	6 73	93 12	16 85	157
158	74 6	16 86	94 12	158
159	7 75	87 15	13 91	159
160	76 7	13 92	88 15	160

	161	162	
	165 164	166 163	1
146	8 – 77	78 – 8	146
147	81 9	10 79	147
148	10 80	82 9	148

Addition to Table II.

	146	149	150	
1	147 148	152 153	151 154	1
2	149 150	146	...... –146	2
3	151 152	 147	 148	3
4	153 154	148	147	4
5	155 156	....	....	5
6	157 158	....	....	6
7	159 160	....	....	7
8	161 162	150	149	8
9	163 164	 151	 152	9
10	165 166	154	153	10
11	167 168	156	155	11
12	169 170	 157	 158	12
13	171 172	160	159	13
14	173 174	...... –155	...... –156	14
15	175 176	 159	 160	15
16	177 178	158	157	16
62	146	 150	...... –149	62
63	 147	151	 153	63
64	148	 154	152	64
65	 149	...... –146	....	65
66	150	....	146	66
67	 151	....	 147	67
68	152	 148	....	68
69	 153	147	....	69
70	154	....	148	70
71	 155	....	....	71
72	156	....	....	72

	146	149	150	
73	 157	....	....	73
74	158	....	....	74
75	 159	....	....	75
76	160	....	....	76
77	 161	149	....	77
78	162	....	150	78
79	 163	....	 151	79
80	164	 152	....	80
81	 165	153	....	81
82	166	....	154	82
83	 167	155	....	83
84	168	....	156	84
85	 169	....	 157	85
86	170	 158	....	86
87	 171	159	....	87
88	172	....	160	88
89	 173	...... −156	....	89
90	174	....	...... −155	90
91	 175	....	 159	91
92	176	 160	....	92
93	 177	157	....	93
94	178	....	158	94
146	 62	....	...... − 2	146
147	63 1	3	 4	147
148	1 64	 4	3	148
149	 2	....	....	149
150	2	....	....	150
151	 3	....	 1	151
152	3	 1	....	152
153	 4	1	....	153
154	4	....	1	154
155	 5	....	....	155
156	5	....	....	156
157	 6	....	....	157
158	6	....	....	158
159	 7	....	....	159
160	7	....	....	160
161	 8	2	....	161
162	8	 2	2	162
163	 9	....	 3	163
164	9	 3	....	164
165	 10	4	....	165
166	10	....	4	166
167	 11	5	....	167
168	11	....	5	168
169	 12	6	 6	169
170	12	 6	....	170
171	 13	7	....	171
172	13	....	7	172
173	 14	...... − 5	....	173
174	14	....	...... − 5	174
175	 15	....	 7	175
176	15	 7	....	176
177	 16	....	....	177
178	16	....	6	178
179	 17	....	....	179
180	17	....	....	180
181	 18	....	....	181
182	18	....	....	182
183	 19	....	....	183
184	19	....	....	184

	161	162	
8	146	...... −146	8
9	 147	 148	9
10	148	147	10
77	...... −146	....	77
78	....	146	78
79	....	 147	79
80	 148	....	80
81	147	....	81
82	....	148	82
163	....	 1	163
164	 1	....	164
165	1	....	165
166	....	1	166

On the Precession of the Equinoxes, supposing the Earth to revolve in a resisting medium.

In my last paper on Physical Astronomy, I gave expressions for the variations of the six constants which enter into the solution of this problem, upon the hypothesis that the body revolves in a medium devoid of resistance.

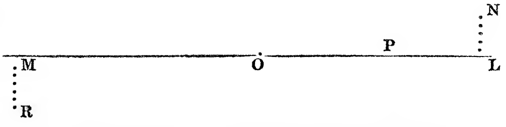
If we suppose a plane to revolve in a resisting medium, about an axis perpendicular to itself, the resistance of the medium can produce no effect, and the phenomena will only be modified in a slight degree by the friction of the plane surface against the medium. If, however, the inclination of the plane on the axis of rotation differs from 90°, the effect of the resistance of the medium becomes sensible, tending to retard the motion of the plane; the effect being greatest when the axis of rotation is parallel to the plane.

This principle is used in some machines, as in self-playing organs, to regulate the motion by means of a vane, of which the inclination to its axis of rotation can be varied at pleasure.

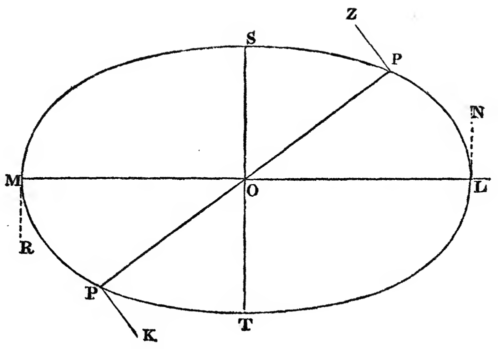
In the case of a sphere, whatever be the direction of the axis of rotation, this effect of the resistance is insensible; and also in the case of a solid of revolution when the axis of rotation coincides with the axis of the figure, but not otherwise. If the difference of the latitude of the axis of rotation from 90° (supposing the equator from which the latitudes are reckoned to coincide with the equator of the figure) be at any time small, the mathematical investigation appears to show, that the effect of the resistance of the medium is to diminish continually this difference. In the case of the earth, this quantity is now insensible; but as the probability is small that this was the case in the first instance, may this circumstance arise from the resistance of a medium of small density acting for a great length of time? and can the change of climate on the surface of the earth, a change of which the probability is indicated by many geological phenomena, be explained in the same manner? It may be remarked, however, that the effect of a resisting medium in diminishing the eccentricities of the orbits of the planets is of the same order, and that these, although for the most part small, are far from having reached zero. The tendency of a resisting medium is also to diminish the major axes of the orbits of the planets; these effects, if they exist, will probably be most sensible in the case of comets, not only on account of their great eccentricity, but also on account of their small density, in the same manner as a flock of any light substance is wafted by the gentlest wind and prevented from reaching the ground. The eccentricity of the orbit of the comet of Halley in 1759 is known with great accuracy, and as its perturbations have been calculated with great care by MM. Damoiseau and de Pontécoulant, the eccentricity which it should have in 1835, when it will again visit this part of space, unless it be affected by a resisting medium, is also known with great precision. It is scarcely probable, however, that any change will be perceptible in one revolution, even if the cause exists; but the succeeding revolutions of this body will no doubt throw light upon this question. The ratio of the change of the semi-major axis to the change of the eccentricity, due to the action of the resisting medium, is known, being a function of the eccentricity, and independent of the constant, which depends upon the density of the medium; this ratio therefore may also tend to elucidate the question, if it can be determined by observation with sufficient accuracy.

Let x', y', z' be the co-ordinates of any point P corresponding to the elementary portion of the surface $\mathrm{d}s$, and referred to axes passing through the centre of gravity and revolving with the body in motion.

Let P be the point of which the co-ordinates are x', y', z', A P the direction of the normal at the point P, B P perpendicular to the axis of instantaneous rotation, and cutting it in B, and C P the direction of motion of the point P. I suppose the resistance of the medium to create a force proportional to $v^2 \cos \mathrm{APC}\, \mathrm{d}s$, acting in the direction of the normal A P upon the point P, v being the velocity of the point P.



Suppose the straight line M O P L to revolve about an axis passing through O, and perpendicular to it, and in the direction L N, the action of the resisting medium will be in the direction N L, on one side only of the line O L, upon all the points P between O and L, and upon all the points between M P it will be in the contrary direction R M, and on the other side of the line.



Now, let L S M T be the section of a cylinder revolving about an axis, passing through O perpendicular to the plane L S M T, and let the cylinder revolve in the direction L N. The action of the resisting medium will be in the direction Z P, perpendicular to O P upon all the points P between L S; and in the contrary direction K P upon all the points, P between T M. These remarks show that in what follows, the integrations must not be made throughout the whole surface of the body revolving: this consideration however does not affect the nature of the results.

The equation to a plane perpendicular to the axis of rotation, and passing through the centre of gravity of the body, is $p x + q y + r z = 0$.

Let the body revolving be a spheroid of which the equation is

$$x^2 + y^2 + z^2 (1 + e^2) = a^2 (1 + e^2)$$

The equation to the tangent plane to the spheroid at the point x, y, z is

$$x x' + y y' + z z' (1 + e^2) = a^2 (1 + e^2)$$

The equations to the planes from whose intersection the line P B results, are

$$* z (q z' - r y') + y (r x' - p z') + z (p y' - q x') = 0$$
$$p x + q y + r z = D$$

D being a constant. The equations to the line P C are

$$x\{r (q z' - r y') - p (p y' - q x')\} + y \{r (r x' - p z') - q (p y' - q x')\} = 0$$
$$x\{q (q z' - r y') - p (r x' - p z')\} + z \{q (p y' - q x') - r (r x' - p z')\} = 0$$

and neglecting p^2, q^2, $p q$,

$$x (q z' - r y') = y (p z' - r x')$$
$$x (q y' + p x') = z (p z' - r x')$$

The equations to the direction of motion of the point P are

$$x (p z' - r x') = y (r y' - q r')$$
$$x (q x' - p y') = z (r y' - q z')$$

Cos. angle, which the direction of motion of P makes with the normal to the surface or cos A P C

$$= \frac{x' (r y' - q z') + y' (p z' - r x') + z' (1 + e^2) (q x' - p y')}{\surd \{(r y' - q z')^2 + (p z' - r x')^2 + (q y - p x')^2\} \surd \{x'^2 + y'^2 + z'^2 (1 + 2 e^2)\}}$$

---

* The notation is the same as p. 20, except that the accents at foot of $x_{\prime}$, $y_{\prime}$, $z_{\prime}$ are omitted.

$$= \frac{e^2 z' (q x' - p y')}{r \sqrt{x'^2 + y'^2} \sqrt{x'^2 + y'^2 + z'^2}} \quad \text{nearly.}$$

The resistance acting in the direction of the normal, and since the velocity $= \sqrt{x'^2 + y'^2} \sqrt{(p^2 + q^2 + r^2)}$ nearly;

$$C \, \mathrm{d} r = 0$$

$$B \, \mathrm{d} q + (A - C) r p \, \mathrm{d} t = \mathrm{d} t \int \frac{\{z' x' - x' z' (1 + e^2)\} e^2 z' (q x' - p y') \sqrt{x'^2 + y'^2} \, \mathrm{d} s \, (p^2 + q^2 + r^2)}{r \{x'^2 + y'^2 + z'^2\}}$$

$$A \, \mathrm{d} p + (C - B) q r \, \mathrm{d} t = \mathrm{d} t \int \frac{\{y' z' (1 + e^2) - z' y'\} e^2 z' (q x' - p y') \sqrt{x'^2 + y'^2} \, \mathrm{d} s \, (p^2 + q^2 + r^2)}{r \{x'^2 + y'^2 + z'^2\}}$$

$$\sin \frac{C - A}{A} (n t + \gamma) \, \mathrm{d} c + c \frac{(C - A)}{A} \cos \frac{C - A}{A} (n t + \gamma) \, \mathrm{d} \gamma$$

$$= - \frac{n \, \mathrm{d} t \, e^4}{A} \int \frac{x' z'^2 (q x' - p y') \sqrt{x'^2 + y'^2} \, \mathrm{d} s}{\{x'^2 + y'^2 + z'^2\}}$$

$$\cos \frac{C - A}{A} (n t + \gamma) \, \mathrm{d} c - c \frac{(C - A)}{A} \sin \frac{C - A}{A} (n t + \gamma) \, \mathrm{d} \gamma$$

$$= \frac{n \, \mathrm{d} t \, e^4}{A} \int \frac{y' z'^2 (q x' - p y') \sqrt{x'^2 + y'^2} \, \mathrm{d} s}{\{x'^2 + y'^2 + z'^2\}}$$

since $\int x'^2 z'^2 \, \mathrm{d} s = \int y'^2 z'^2 \, \mathrm{d} s$

$$\mathrm{d} c = - \frac{n \, \mathrm{d} t \, e^4 c}{A} \int \frac{x'^2 z'^2 \sqrt{x'^2 + y'^2} \, \mathrm{d} s}{\{x'^2 + y'^2 + z'^2\}} + \frac{n \, \mathrm{d} t \, e^4}{2 A} \sin 2 \frac{(C - A)}{A} (n t + \gamma) \int \frac{x' y' z'^2 \sqrt{x'^2 + y'^2} \, \mathrm{d} s}{\{x'^2 + y'^2 + z'^2\}}$$

neglecting the term which is periodic,

$$\mathrm{d} c = - n c \frac{e^4 \, \mathrm{d} t}{A} \int \frac{x'^2 z'^2 \sqrt{x'^2 + y'^2} \, \mathrm{d} s}{\{x'^2 + y'^2 + z'^2\}}$$

Let $\int \frac{x'^2 z'^2 \sqrt{x'^2 + y'^2} \, \mathrm{d} s}{\{x'^2 + y'^2 + z'^2\}} = D$

D being a positive quantity.

$\mathrm{d} c = - \frac{n D c e^4 \, \mathrm{d} t}{A} \qquad e^{\frac{1}{c}} = \frac{n D e^4 t}{A}$, e being the base of Naperian logarithms.

When t is infinite $c = 0$; hence the latitude of the axis of instantaneous rotation increases until it reaches 90°, which is its limit.

Having determined the variations of c, γ and n by means of the above equations, the variations of the other constants ω, ψ_0 and φ_0 may be determined from the equations

$$p \, \mathrm{d} t = \sin \varphi \sin \theta \, \mathrm{d} \psi - \cos \varphi \, \mathrm{d} \theta$$
$$q \, \mathrm{d} t = \cos \varphi \sin \theta \, \mathrm{d} \psi + \sin \varphi \, \mathrm{d} \theta$$
$$r \, \mathrm{d} t = \mathrm{d} \varphi - \cos \theta \, \mathrm{d} \psi$$